MATHEMATICS 8

Statistics and Probability

Distance Learning

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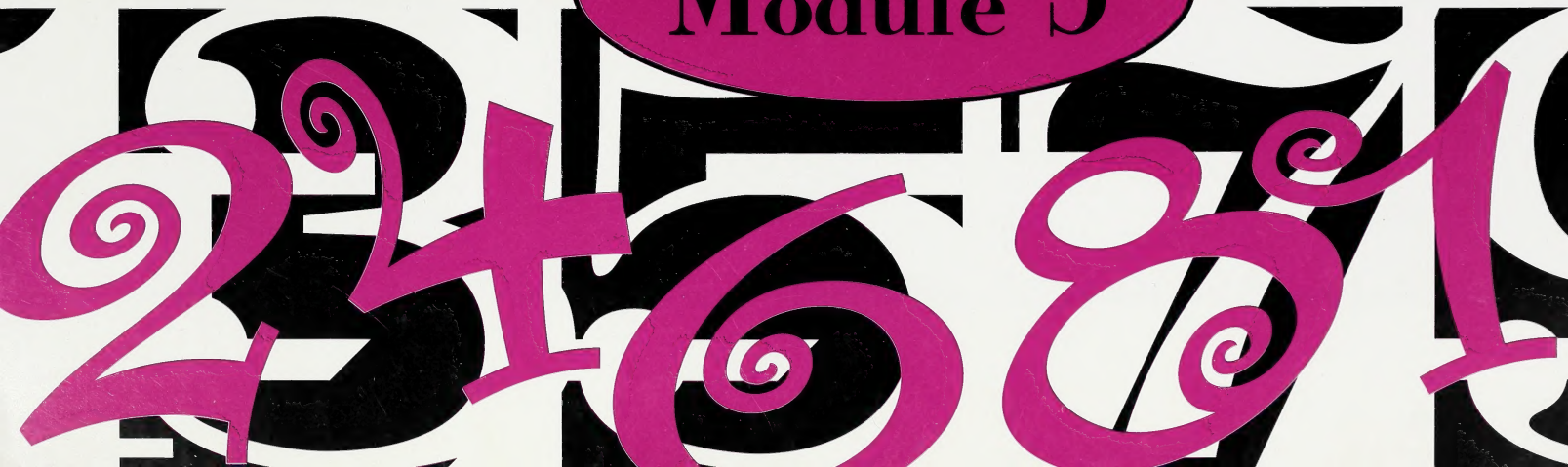
Module 5



Mathematics
Grade: 8
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Mathematics 8

Module 5

Statistics and Probability

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Student Module Booklet
Module 5
Statistics and Probability
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

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http://ednet.edc.gov.ab.ca/ltb

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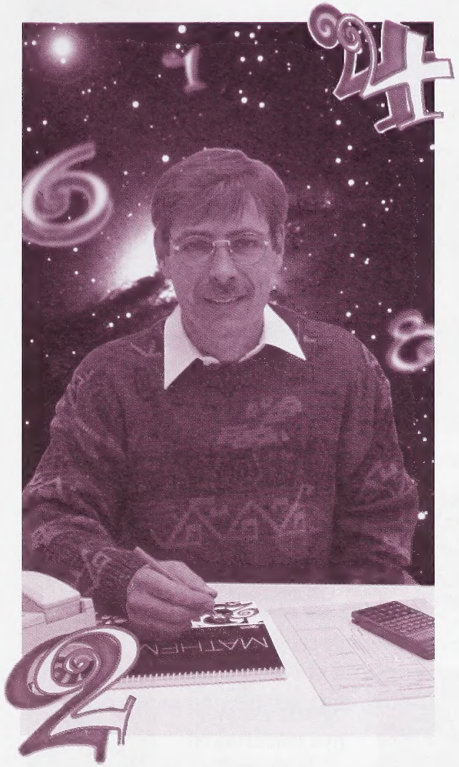
Welcome



Welcome to Module 5. We hope you'll enjoy your study of Statistics and Probability.

Mathematics 8 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

Mathematics 8

The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

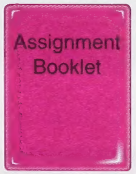
- Use computer software.

- Use a scientific calculator.

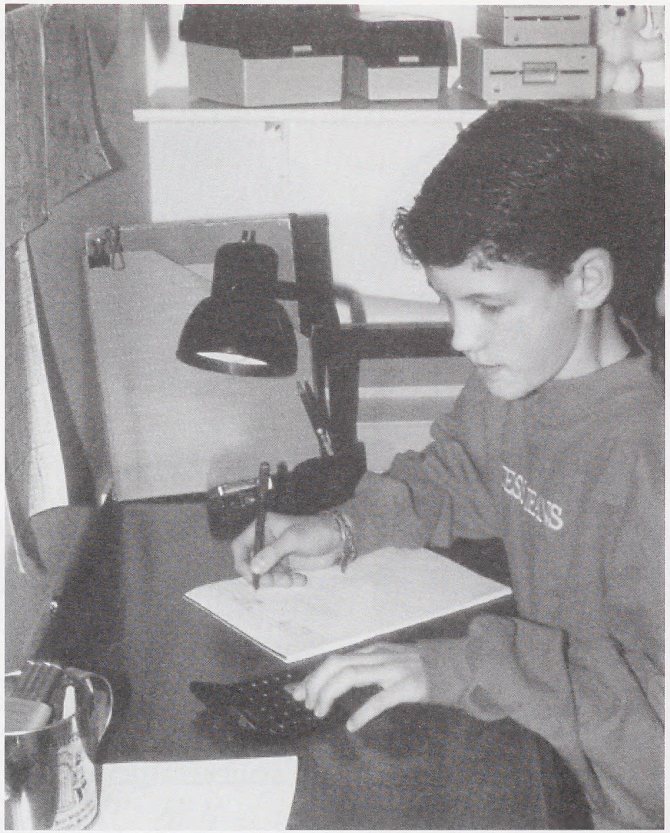
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

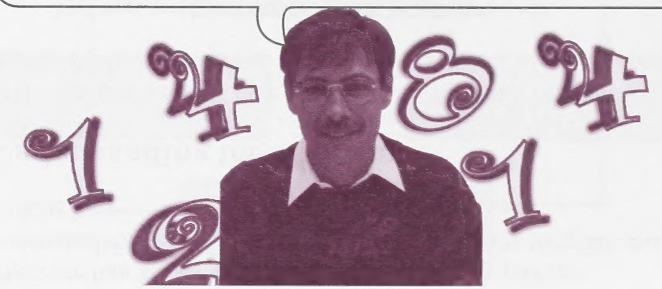
In Mathematics 8, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all of you will be able to view the video programs and use a calculator, and that most of you will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator, and draw figures and graphs by hand.



Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge. Watch for these icons.

This icon is a cue that the problem will be related to the topic of the activity.

This icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- using objects
- using diagrams
- making a table
- working backwards
- using elimination
- using truth tables
- using an equation
- changing your point of view
- making an organized list
- using Venn diagrams
- simplifying a problem
- guessing, checking, and revising
- finding and applying a pattern
- acting out a problem

Note: The Appendix in Module 6 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Module 6 Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

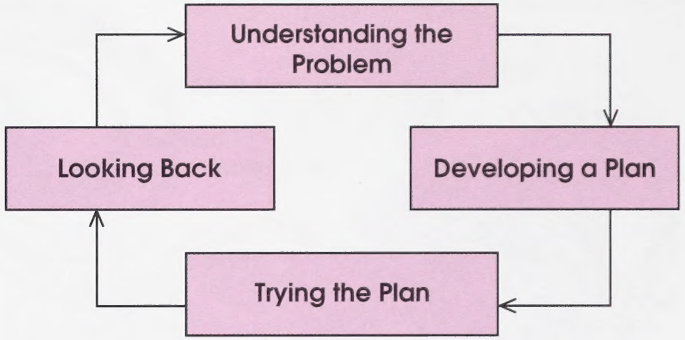
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

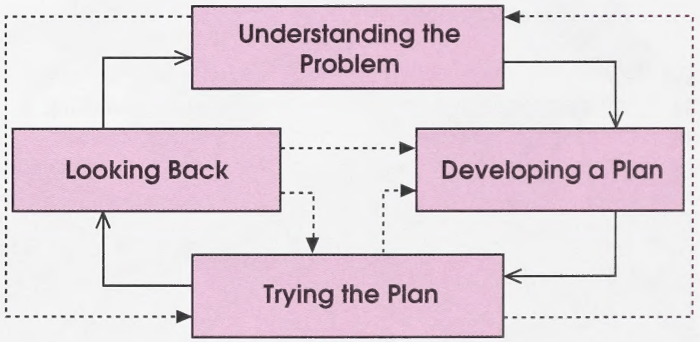
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



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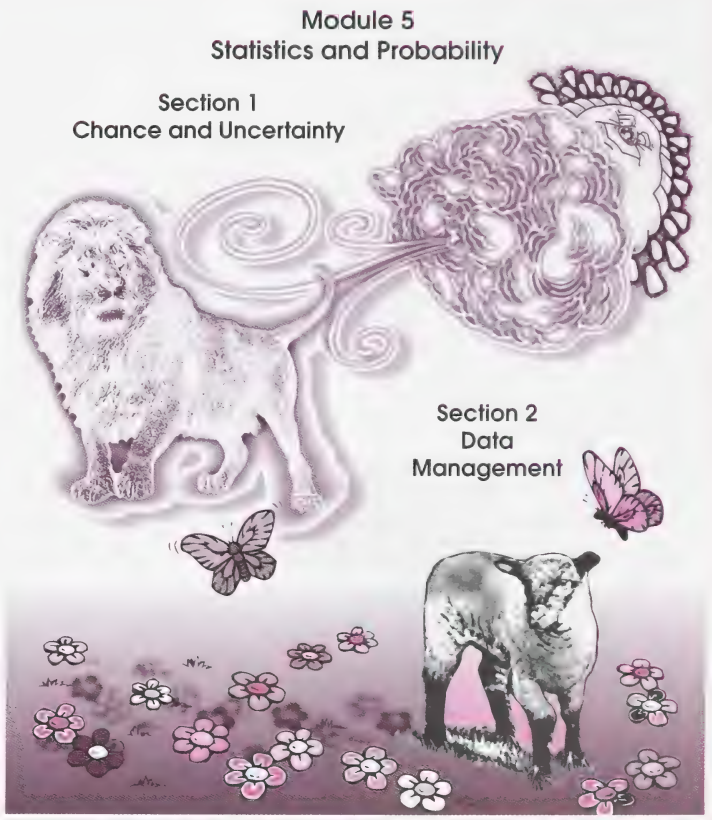
Module Overview

Do you enjoy winter? Is there any way to forecast how severe or mild the winter will be in your area? According to folklore there are several indicators. Some say that sundogs (spots that sometimes appear on either side of the sun) are a sure sign of cold weather. Others think that if you see geese with white breast feathers in September, the winter will be severe; if the geese have dark breast feathers in September, the winter will be mild. Still others insist that if March comes in like a lamb, it will go out like a lion. In other words, if March 1 is blustery and cold, March 31 will be warm and sunny.

How do you think these weather signs originated? Do you think they have any basis in fact? How do the authors of books like *The Old Farmer's Almanac* make their predictions? How do the weather forecasters determine the likelihood of rain, snow, or sunshine?

In this module you will investigate chance and uncertainty. You will also explore data management.



Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	**35 marks**
Section 2 Assignment	**40 marks**
Final Module Assignment	**25 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

If you are working on a computer managed learning (CML) terminal, you will have a module test as well as a module assignment.

Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

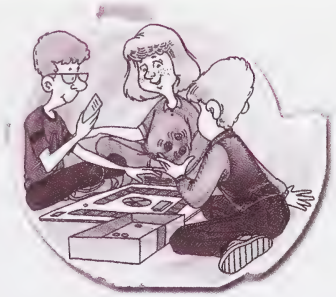
Section 1: Chance and Uncertainty

The insurance business uses statistics and probability to predict which group of drivers is most likely or least likely to have motor vehicle accidents. Statistically, young men have the most accidents; therefore, their insurance rates are usually higher than other groups of drivers. There are data to show that having driver education decreases the likelihood of having a motor vehicle accident; hence, insurance companies usually give discounts to graduates of driver-education programs.

Statistics and probability are used in many fields ranging from business to medicine. For example, governments and environmental groups use mathematics to assess the risk of various undertakings such as the construction of a nuclear-power plant, a hazardous-waste plant, or a mine. Doctors use mathematics to predict the likelihood that an operation or treatment plan will be successful.

In this section you will use games of chance to review what you already know about probability and to extend your knowledge. You will also use samples, statistics, and simulations to make predictions.

Activity 1: Games of Chance



Do you enjoy playing games? You have probably discovered that games such as *Monopoly®*, *Sorry®*, *Scattergories®*, *Boggle®*, and the *SCRABBLE® Brand Crossword Game* involve an element of chance.

In this activity you will use games of chance to review what you already know about chance and uncertainty and to extend your knowledge.

When you play word games such as *Boggle®*, *Scattergories®*, and the *SCRABBLE® Brand Crossword Game*, you quickly realize that some letters are used more frequently than others in English words.

The following chart shows the **relative frequency** of letters in a large sample of material written in English.

Relative frequency is the ratio of the number of times something has occurred to the number of times it could occur.

Analysis of Letters in Sample

Letter	Relative Frequency	Letter	Relative Frequency
A	0.082	N	0.070
B	0.014	O	0.080
C	0.028	P	0.020
D	0.038	Q	0.001
E	0.130	R	0.068
F	0.030	S	0.060
G	0.020	T	0.105
H	0.053	U	0.025
I	0.065	V	0.009
J	0.001	W	0.015
K	0.004	X	0.002
L	0.034	Y	0.020
M	0.025	Z	0.0007

Use the chart to answer the following questions.

1. a. What is the most frequently used vowel in the English language? **Hint:** The vowels include, A, E, I, O, U, and Y.

b. What are the two least frequently used vowels in the English language?

c. What are the three most frequently used consonants in the English language?

d. What are the three least frequently used consonants in the English language?

2. In the *SCRABBLE® Brand Crossword Game*, players draw tiles at random from a bag. Of the 100 tiles in the game, some tiles have letters on them and some are blank.

Following is a chart showing the distribution of the letters and blanks on the tiles.

Distribution of Tiles

Letter	Number of Tiles
A	9
B	2
C	2
D	4
E	12
F	2
G	3
H	2
I	9
J	1
K	1
L	4
M	2
N	6

Letter	Number of Tiles
O	8
P	2
Q	1
R	6
S	4
T	6
U	4
V	2
W	2
X	1
Y	2
Z	1
Blank	2

Do you think the relative frequency of letters in English words was used in determining the number of tiles with each letter? Why or why not?

3. In the television show *Wheel of Fortune*, contestants guess the letters in secret messages (names, phrases).

a. Watch an episode of *Wheel of Fortune* and do the following tasks.

- In a table like this, record the frequency of the letters in the guesses of the contestants. Afterwards, calculate the relative frequency of each letter.

Letters Contestants Guessed	Tally	Frequency	Relative Frequency
a			
b			
c			

- In a table like this, record the frequency of the letters in the secret messages. Afterwards, calculate the relative frequency of each letter.

Letters in Secret Messages	Tally	Frequency	Relative Frequency
a			
b			
c			

b. Compare the relative frequency of the letters the contestants guessed to the relative frequency in the large sample. Did the contestants guess the most frequently used letters more often than the least frequently used letters?

c. Compare the relative frequency of the letters in the secret messages to the relative frequency in the large sample. Did the show choose messages with less frequently used letters?

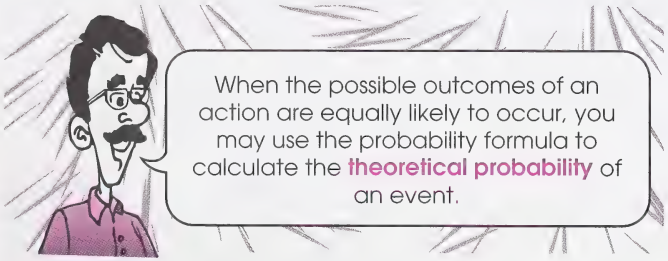
Check your answers by turning to the Appendix.

Use the Internet to play *Wheel of Fortune* on-line. This is the uniform resource locator (URL) of the site.

http://www.wheelfortune.com:80/

You can also try playing games such as *SCRABBLE®* on the Internet at the Hasbro home page. This is Hasbro's URL:

http://www.hasbro.com/

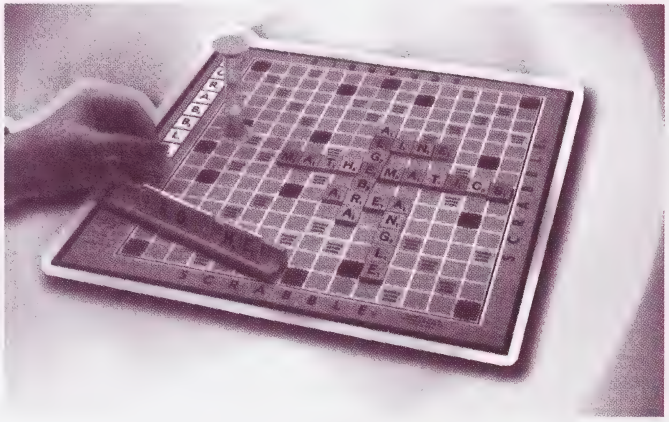

When the possible outcomes of an action are equally likely to occur, you may use the probability formula to calculate the theoretical probability of an event.

The theoretical probability of an event is the ratio of the number of **favourable outcomes** to the number of **possible outcomes**.

Favourable outcomes are what you want to happen. Possible outcomes are what could happen.

Example 1

In the *SCRABBLE® Brand Crossword Game* there are 100 tiles. Of these, there are 9 tiles with A, 12 with E, 9 with I, 8 with O, 4 with U, and 2 with Y.



If you draw a tile at random from a bag containing all the tiles, what is the probability that you will draw a vowel?

Solution

Step 1: Find the number of favourable outcomes.

$$9+12+9+8+4+2=44$$

There are 44 favourable outcomes.

Step 2: Find the number of possible outcomes.

There are 100 possible outcomes.

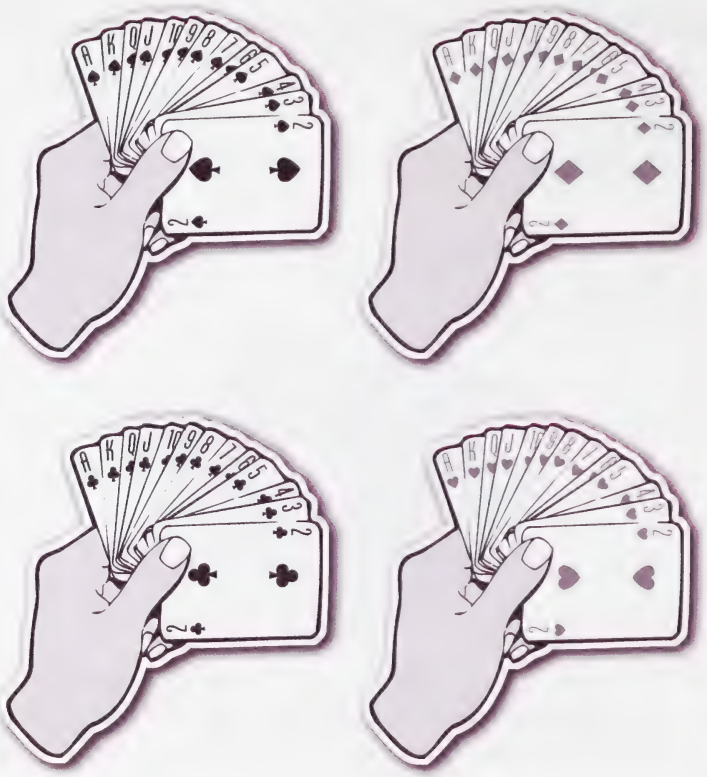
Step 3: Calculate the theoretical probability of drawing a vowel.

$$P(\textit{vowel}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{44}{100}$$
$$= \frac{11}{25}$$

The probability of drawing a vowel is $\frac{11}{25}$.

4. A standard deck of cards has 52 playing cards. There are four suits—clubs (♣), diamonds (♦), hearts (♥), and spades (♠). The clubs and spades are black cards. The diamonds and hearts are red cards.

The cards in each suit are labelled 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A. The jack (J), queen (Q), and king (K) of each suit are face cards. The ace (A) of each suit is usually considered to be the highest card.



Suppose you draw one card at random from a shuffled (mixed) deck of cards. **Hint:** Assume the aces are the highest cards.

a. What is the probability of drawing the ace of spades?
b. What is the probability of drawing a queen?
c. What is the probability of drawing a diamond?

d. What is the probability of drawing a black card?
e. What is the probability of drawing a face card?
f. What is the probability of drawing a card lower than an 8?
g. What is the probability of drawing a card higher than an ace?

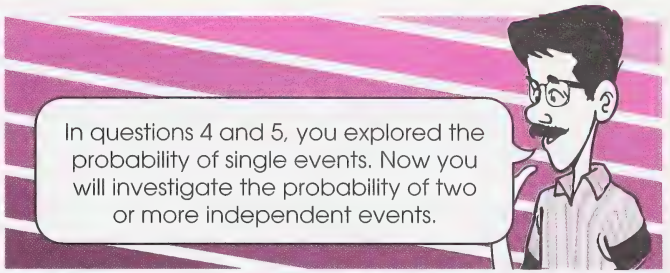
5. In the game *Scattergories*® a 20-sided die is rolled. The sides of the die are labelled A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, R, S, T, W.

a. What is the theoretical probability of rolling the letter L?

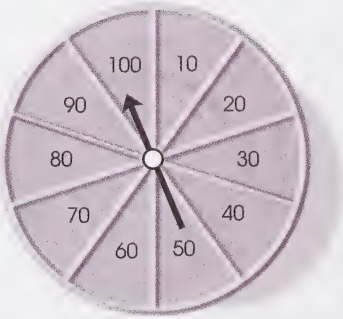
b. What is the theoretical probability of rolling the letter Q?

c. What is the theoretical probability of rolling a vowel?
Hint: The vowels are A, E, I, O, U, and Y.

Check your answers by turning to the Appendix.


In questions 4 and 5, you explored the probability of single events. Now you will investigate the probability of two or more independent events.

6. In the television game show *The Price Is Right*, contestants spin a wheel to determine if they will be able to bid on the showcase at the end of the show. The wheel looks similar to this:



Players have two spins, but may choose to spin only once. The player who gets closest to 100 without going over is chosen for the showcase.

Use the simplified version of the wheel to answer the following questions.

a. Suppose the first contestant gets 100 in two spins. Complete a table like this to show the possible scores the second contestant could spin on one or two spins in this simplified version. **Note:** If the player gets 100 on the first spin, the player would not spin again. If the player gets less than 100 on the first spin, the player must spin again. The first and last columns have been filled for you.

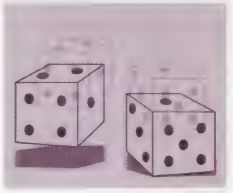
Sums		First Spin									
		10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
Second Spin	**10**	20									100
	20	30									100
	30	40									100
	40	50									100
	50	60									100
	60	70									100
	70	80									100
	80	90									100
	90	100									100
	100	over									100

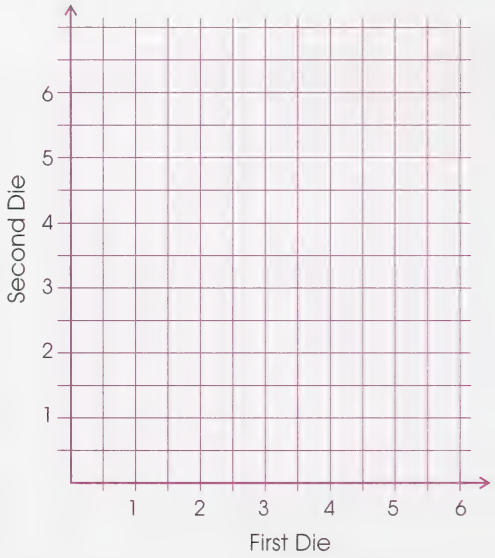
b. What is the probability the second contestant will score 100?

c. What is the probability the second contestant will go over 100?

7. In the game of *Monopoly*®, players sometimes land on a square labelled "Jail." To get out of jail, players must have either a card entitled "get out of jail free" or they must roll "doubles" on a pair of dice. "Doubles" means the same number on each die.

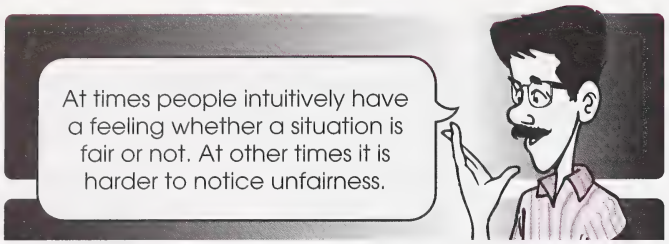
a. Complete a graph like this to show the possible outcomes that can be rolled on a pair of dice. For example, rolling 2 on the first die and 1 on the second die would be shown as $(2,1)$.





b. What is the probability of rolling "doubles"?

Check your answers by turning to the Appendix.


At times people intuitively have a feeling whether a situation is fair or not. At other times it is harder to notice unfairness.

8. In the game "Rock, Scissors, Paper," hand positions are used to represent a rock, a pair of scissors, and a sheet of paper.

The game requires two players. At a signal each player puts out a hand representing one of the objects. The rules are that paper beats rock, rock beats scissors, and scissors beat paper. A player scores a point for winning a round. The winner is the player with the most points after 12 rounds.

a. Play "Rock, Scissors, Paper" with a friend. Record the number of points each person earns in the 12 rounds. Does the game appear to be fair? Does either player seem to have an unfair advantage?

b. Complete a table like this to show all the possible winners in a round of "Rock, Scissors, Paper." The first column has been completed for you as an example.

Winner		Player B's Action		
		Rock	Scissors	Paper
Player A's Action	Rock	tie		
	Scissors	Player B wins.		
	Paper	Player A wins.		

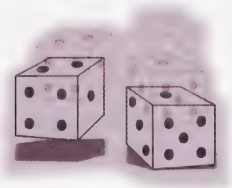
c. What is the theoretical probability of Player A winning a round?

d. What is the theoretical probability of Player B winning a round?

e. What is the theoretical probability there will be a tie in the round?

f. Based on the theoretical probabilities, is the game fair? Why or why not?

9. The game "Dice Fractions" requires two players. You will need 2 standard dice and paper and pencil to keep score.



Following are the rules for the game.

How to Play "Dice Fractions"

- Decide who will be Player A and who will be Player B.
- Choose the starting player by rolling the dice. The player with the higher number goes first.
- The starting player rolls the dice and forms a fraction equal to or less than 1 with the two top faces of the dice. The smaller number goes in the numerator and the larger number goes in the denominator.

  Example: $= \frac{4}{6}$

  If the fraction is **not** in simplest form, Player A scores a point. If the fraction is in simplest form, Player B scores a point. **Note:** A fraction is in simplest form if the numbers in the numerator and denominator are the smallest whole numbers possible.
- Alternate turns for 12 rounds. The winner is the person with more points at the end of the 12 rounds.

a. Play "Dice Fractions" with a friend. Record the number of points each person earns in the 12 rounds. Does the game appear to be fair? Does either player seem to have an unfair advantage?

b. Complete a table like this to show all the possible fractions that can be made in the game. **Note:** The first row has been completed for you as an example.

Fractions		Second Die					
		1	2	3	4	5	6
First Die	1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
	2						
	3						
	4						
	5						
	6						

c. Use the table showing all the possible fractions to complete a table like this to show the possible ways each player can score a point in a round of "Dice Fractions." **Note:** The first row has been completed for you as an example.

Player Who Scores a Point		Player B					
		1	2	3	4	5	6
Player A	1	A	B	B	B	B	B
	2						
	3						
	4						
	5						
	6						

d. What is the theoretical probability of Player A scoring a point in a round of the game?

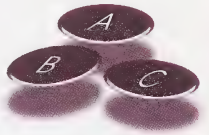
e. What is the theoretical probability of Player B scoring a point in a round of the game?

f. What is the theoretical probability that neither Player A nor Player B will score a point in the round?

g. Based on the theoretical probabilities, is the game fair? Why or why not?

10. The game "ABC" requires two players. You will need the following:

- one chip with an A side and a B side
- one chip with an A side and a C side
- one chip with a B side and a C side
- paper and pencil to keep score



Note: You may use bingo chips, poker chips, coins, or checkers. Label the sides of the "chips" with a marker.

How to Play "ABC"

- Decide who will be Player 1 and who will be Player 2.
- Choose the starting player by flipping one of the chips.
- The starting player flips all three chips at the same time. Player 1 scores a point if the letters on two of the chips match. Player 2 scores a point if the letters on all three chips are different.
- Alternate turns for 12 rounds. The winner is the player with more points after 12 rounds.

a. Play "ABC" with a friend. Record the number of points each person earns in the 12 rounds. Does the game appear to be fair? Does either player seem to have an unfair advantage?

b. Complete a tree like this to list all the possible outcomes.

Chip 1	Chip 2	Chip 3	Outcome	Scorer
A	A	B	(A, A, B)	Player 1
		C	(A, A, C)	Player 2
B				

c. What is the probability that Player 1 scores a point in a round of the game?

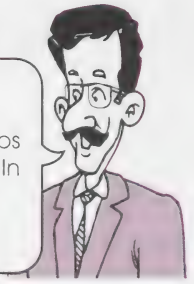
d. What is the probability that Player 2 scores a point in a round of the game?

e. What is the probability that there is a tie for the round?

f. Based on the theoretical probabilities, is the game fair? Why or why not?

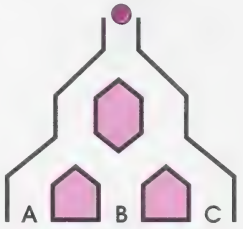
Check your answers by turning to the Appendix.

One of the games played on the television show *The Price Is Right* is "Plunko." In this game a contestant drops chips into the top of a vertical network. In this part of the activity you will solve problems involving chance and networks.



Example 2

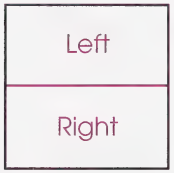
If you drop a chip into the top of this vertical network, what is the probability that it will land at B?



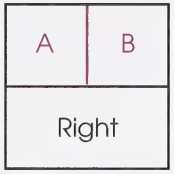
Solution

Step 1: Draw a square to help you visualize the problem.

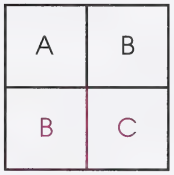
Step 2: At the first branch in the network, the chip is equally likely to go into the left path or the right path. Therefore, divide the square into two equal parts and label the parts "Left" and "Right."



Step 3: Suppose that at the first branch, the chip goes into the left path. This path splits into two more paths. One path leads to A and the other leads to B. The chip is equally likely to go into either of these paths. Therefore, divide the region for the left path (see Step 2) into two equal parts and label the parts "A" and "B."



Step 4: Suppose that at the first branch, the chip goes into the right path. This path splits into two more paths. One path leads to B and the other path leads into C. The chip is equally likely to go into either of these paths. Therefore, divide the region for the right path (see Step 3) into two equal parts and the label the parts "B" and "C."



Step 5: Use the diagram in Step 4 to calculate the probability of the chip landing at B.

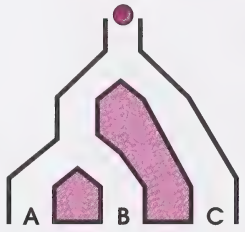
The diagram shows that there are 2 favourable outcomes out of 4 possible outcomes.

$$P(B) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

The probability of the chip landing at B is $\frac{1}{2}$.

Example 3

If you drop a chip into the top of this vertical network, what is the probability that it will land at C?

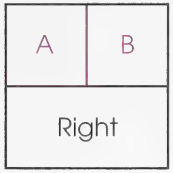


Solution

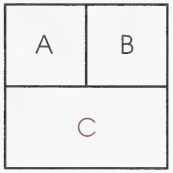
Step 1: Draw a square to help you visualize the problem.

Step 2: At the first branch in the network, the chip is equally likely to go into the left path or the right path. Therefore, divide the square into two equal parts and label the parts "Left" and "Right."

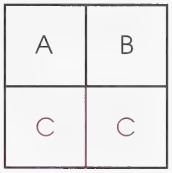
Step 3: Suppose that at the first branch, the chip goes into the left path. This path splits into two more paths. One path leads to A and the other path leads to B. The chip is equally likely to go into either of these paths. Therefore, divide the region for the left path (see Step 2) into two equal parts and label the parts "A" and "B."



Step 4: Suppose that at the first branch, the chip goes into the right path. This path leads to C. Therefore, label the region for the right path (see Step 3) as "C."



Step 5: The part labelled C has twice the area of the parts labelled A and B. Therefore, to make comparison easier, divide the region labelled C into two equal parts and label each part "C."



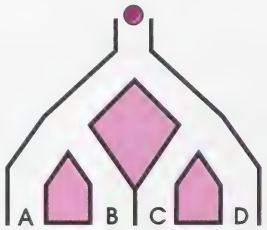
Step 6: Use the diagram in Step 5 to calculate the probability of the chip landing at C.

The diagram shows that there are 2 favourable outcomes out of 4 possible outcomes.

$$P(C) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

The probability of the chip landing at C is $\frac{1}{2}$.

11. Suppose you drop a chip into the top of this vertical network.



a. What is the probability that the chip will land at A?
b. What is the probability that the chip will land at B?

12. Suppose you drop a chip into the top of this vertical network.

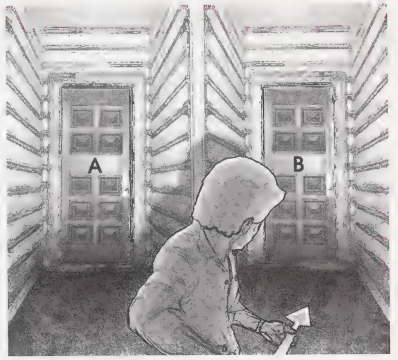
a. What is the probability the chip will land at A?
b. What is the probability the chip will land at B?

Check your answers by turning to the Appendix.

Now Try This

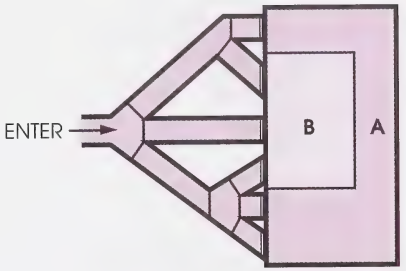
Use a problem-solving strategy to answer the following question.

13. The king and queen of a medieval kingdom arranged a marriage for their daughter to Prince Cuthbert from a neighbouring kingdom. The princess accepted this plan without enthusiasm. A short time before the proposed wedding day she met Reynaldo—handsome, clever, romantic, but only a peasant. Their love developed quickly and secretly, but inevitably the king learned of their relationship.



Irate, the king ordered that Reynaldo be thrown into a room full of tigers. But in response to his daughter's pleas, he offered a compromise: Reynaldo would walk through a maze, each path leading to one of two rooms. While the hungry tigers waited in one room, the hopeful princess would wait in the other room. If Reynaldo entered the latter room, he and the princess could marry.

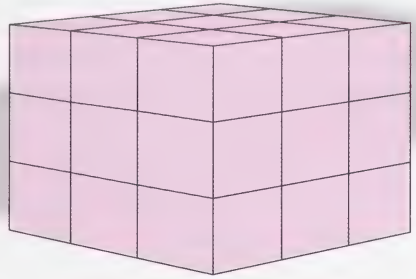
The king showed the princess a map like this one of the maze and let her decide in which room to wait.



Which room is Reynaldo more likely to enter, A or B? Remember that Reynaldo does not have a copy of the map and can only guess which paths to follow.[1]

[1] Albert P. Shulte and James R. Smart, eds., "Marriage by Chance," in *Teaching Statistics and Probability*, 1981, 136-137. Reprinted by permission of the National Council of Teachers of Mathematics.

14. You are given 27 small cubes and you arrange the cubes to form a large cube. Then you paint the six faces of the large cube.



After the paint dries, you take all 27 small cubes and put them in a large paper bag. You shake the cubes so that they are well mixed. You reach into the bag and pick one cube at random.

What is the probability that the cube you pick has exactly two painted faces?

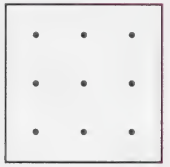
15. Six coins are in a paper bag. At least one penny, one nickel, and one dime are in the bag. There are more nickels than dimes and more dimes than pennies.

If a coin is chosen at random from the bag, what is the probability of selecting a penny?

Check your answers by turning to the Appendix.

Use a problem-solving strategy to answer each of the following questions.

16. In a diagram like this draw two squares so that each of the dots is separated from the other dots.

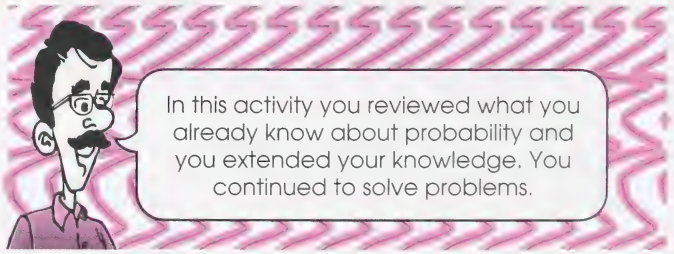


17. Jane has 3 sisters and 2 brothers.

a. How many sisters does Jane's eldest sister have?
b. How many sisters does Jane's eldest brother have?

18. A mystery guest states, "I have two more sisters than brothers. My **eldest sister** also has two more sisters than brothers. How many more sisters than brothers does my **eldest brother** have?"

Check your answers by turning to the Appendix.


In this activity you reviewed what you already know about probability and you extended your knowledge. You continued to solve problems.

Activity 2: Samples and Statistics

When cooks make soup, they usually taste a spoonful of the soup to check the flavour. This **sample** helps the cook decide if the soup has the right amount of seasoning.

In statistics, the set of people, animals, or things being studied is called the **population**. The part of the population that is surveyed or tested is referred to as the sample.

If a sample is selected properly, it can be used to draw conclusions about the population.

Example 1

A manufacturer produces boxes of sterile bandages. To check the quality of the production and correct the manufacturing process if necessary, a sample of boxes are selected and the bandages in these boxes are tested by quality-control inspectors.

On a particular day 20 000 boxes of bandages are produced and 400 boxes are selected at random to be tested. In the sample, 3 boxes are found to have substandard bandages.

How many boxes in the total production run are likely to have substandard bandages?

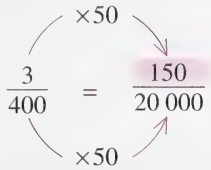
Solution

Method 1: Using a Proportion

Step 1: Use this reasoning. The ratio of the number of boxes with substandard bandages to the number of boxes produced will be the same in the sample and the total production run. Write a proportion.

$$\frac{\text{boxes with substandard bandages}}{\text{boxes produced}} \qquad \frac{3}{400} = \frac{\quad}{20\,000}$$

Step 2: Find the missing term in the proportion.



Of the 20 000 boxes produced, about 150 are likely to have substandard bandages.

Method 2: Using a Fraction

Step 1: Use this reasoning. Write the fraction of the sample that has substandard bandages.

In the sample, $\frac{3}{400}$ of the boxes have substandard bandages.

In the total production run, the same fraction of boxes is likely to have substandard bandages.

Step 2: Calculate the number of boxes in the total production run that are likely to have substandard bandages.

$$\frac{3}{400} \times 20\,000 = 150$$

Of the 20 000 boxes produced, about 150 are likely to have substandard bandages.

Method 3: Using a Percent

Step 1: Use this reasoning. Write the percentage of the sample that has substandard bandages.

$$\frac{3}{400} = \frac{0.75}{100} = 0.75\%$$

In the sample, 0.75% of the boxes have substandard bandages.

In the total production run, the same percentage of boxes is likely to have substandard bandages.

Step 2: Calculate the number of boxes in the total production run that are likely to have substandard bandages.

$$0.75\% \times 20\,000 = 0.0075 \times 20\,000$$
$$= 150$$

Of the 20 000 boxes produced, about 150 are likely to have substandard bandages.

1. A junior high school with 300 students is holding elections for student council. There are four candidates for president—Tiffany Chan, Marc Piard, Frank Wilson, and Gillian Kowalchuk.

To predict which candidate is most likely to be elected president, the school newspaper conducted a poll. A sample of 30 students was selected at random.

The following table shows the results of the poll.

Election Survey		
Candidate	**Tally**	**Frequency**
Tiffany Chan	𝍸 I	6
Marc Piard	𝍸 II	7
Frank Wilson	IIII	4
Gillian Kowalchuk	𝍸 𝍸 III	13

a. How many of the 300 students at the school are likely to vote for Tiffany Chan? Marc Piard? Frank Wilson? Gillian Kowalchuk?

b. Which student is most likely to be elected president?

2. Each day the junior high school classes in a city have newspapers delivered to them. To predict which section of the newspaper the students are most likely and least likely to read, the local newspaper conducted a survey.

Out of the 12 000 junior high school students in the city, a sample of 200 students was selected at random. The students in the sample were asked which section or sections of the newspaper they read.

The following chart gives the results of the survey.

Newspaper Reading Habits of Junior High Students	
Newspaper Section	**Percent**
Local and Regional News	34
National News	19
International News	17
Editorial/Letters to the Editor	6
Business	3
Sports	38
Arts and Entertainment	42
Comics	56
Homemaking	8
Classified Ads	12

a. Which section of the newspaper is the total population of junior high students most likely to read?

b. Which section of the newspaper is the total population of junior high students least likely to read?

c. Of the 12 000 junior high students in the city, how many are likely to read the Sports section?

d. Of the 12 000 junior high students in the city, how many are likely to read the Editorial/Letters to the Editor section?

3. There are four blood types—A, B, AB, and O. Each type of blood is also classified by an Rh factor—positive or negative.

To investigate the distribution of blood types, a survey was conducted in a community of 50 000 people. A sample of 500 people was selected at random and the blood type of each person in the sample was determined.

The findings are shown in the following table.

Blood Survey	
Blood Type	**Percent**
A Positive	34
A Negative	6
B Positive	10
B Negative	2
AB Positive	4
AB Negative	1
O Positive	37
O Negative	6

a. Based on the survey results, the rarest blood type is AB negative. Of the 50 000 people in this community, how many are likely to have this blood type?

b. Based on the survey, the most common blood type is O positive. Of the 50 000 people in this community, how many are likely to have this blood type?

Check your answers by turning to the Appendix.

Use the Internet to discover more about blood types. You may find the following site helpful.

http://sin2.fi.edu/biosci2/blood/types.html

To monitor the well-being of an animal species, wildlife officers keep records of the number of animals in a population. A sampling method known as the **capture-recapture** method is often used.

A capture-recapture method is a sampling method used to estimate the number of wild animals in a region.

Example 2

In order to estimate the deer population in a provincial park, wildlife officers used the capture-recapture method.

The officers selected a sample of 50 deer, tagged the deer, and then released them.

One month later the officers selected a sample of 100 deer. The officers found that in this sample 5 deer had been tagged the month before.

Estimate the total number of deer in the park.

Solution

Step 1: Use this reasoning.

In the first sample, 50 deer were tagged and released.

In the second sample, 5 out of 100 deer had been tagged the month before.

The ratio of the tagged deer to the total number of deer is the same in the second sample and the total population. Write a proportion.

$$\frac{\text{number of tagged deer}}{\text{total number of deer}} \qquad \frac{5}{100} = \frac{50}{\square}$$

Step 2: Simplify the proportion.

$$\frac{1}{20} = \frac{50}{\square} \qquad \frac{\overset{1}{\cancel{5}}}{\underset{20}{\cancel{100}}}$$

Step 3: Find the missing term in the proportion.

$$\frac{1}{20} \overset{\times 50}{=} \frac{50}{1000}$$

There are about 1000 deer in the park.

4. In order to estimate the trout population in a small lake, biologists used a net to capture a sample of 200 trout. They tagged the dorsal fin of each captured trout and then released the trout.

One week later the biologists used a net to capture a second sample of 250 trout. The biologists discovered that 40 trout had been tagged the week before.

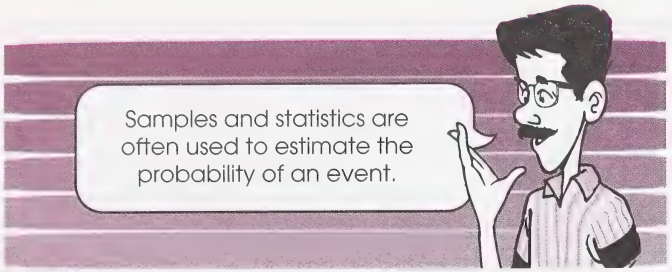
Estimate the total number of trout in the lake.

5. Last month a group of environmentalists in a certain region captured and banded 150 golden eagles.

This month the group captured 200 golden eagles and found that 125 had been banded the previous month.

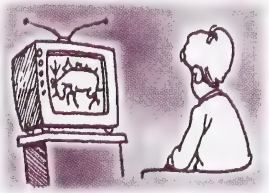
Estimate the number of golden eagles in the region.

Check your answers by turning to the Appendix.



Example 3

In the last month, Frank watched 80 commercials on television. He noticed that 60 of the commercials were advertisements for foods or beverages.



What is the probability that the next commercial Frank watches is a food or beverage advertisement?

Solution

In the sample, 60 out of 80 of the commercials were advertisements for foods or beverages. That is a relative frequency of 0.75.

$$\therefore\ P(\textit{food or beverage advertisement}) = 0.75$$

The probability that the next commercial Frank watches is a food or beverage advertisement is 0.75.

Example 4

Louisa plays in a baseball league. This season she has been up to bat 97 times.

Following are her individual batting statistics for the season.

Outcome	Frequency
Strikeouts	38
Singles	33
Doubles	8
Triples	3
Home Runs	1
Walks	14

What is the probability that Louisa will strike out the next time she is up to bat?

Solution

Louisa has struck out 38 times out of 97 times at bat. That is a relative frequency of about 0.39.

$$\therefore\ P(\textit{strikeout}) \doteq 0.39$$

The probability that Louisa will strike out the next time is 0.39.

6. Gerald has played 96 video games in the last three months. He has won 24 of these games.

   What is the probability that Gerald will win the next video game he plays?

7. In a survey, 24 out of 30 people preferred hamburgers to hot dogs.

   At a barbecue, guests have a choice of a hamburger or a hot dog. What is the probability that a guest will select a hamburger?

8. Elaina is in a bowling league. During the past year she has played 90 games. In 12 of these games, she had a score over 200.

   What is the probability that Elaina will have a score over 200 in the next game she plays?

9. Raphael surveyed 50 families at random to discover what home electronic equipment they owned. The results are shown in the following table.

Families with Home Electronic Equipment		
Type	**Tally**	**Frequency**
Audiocassette Player	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40
Compact Disc Player	𝍸 𝍸 𝍸 I	16
Radio	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 IIII	49
Television	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	48
Video Camera	𝍸	5
Videocassette Player	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40

   What is the probability that a family in the community owns a videocassette player?

Check your answers by turning to the Appendix.

Sometimes **chance** is used to express probability.

The chance of an event happening is the probability of the event happening. Chance is usually expressed as a percent.

Example 5

Suppose there is a 70% chance of rain. What is the probability of rain?

Solution

$$70\% = \frac{70}{100}$$
$$= \frac{7}{10}$$

The probability of rain is $\frac{7}{10}$.

Example 6

Suppose the probability of rain is $\frac{2}{5}$. What are the chances of rain?

Solution

$$\frac{2}{5} = \frac{40}{100}$$
$$= 40\%$$

There is a 40% chance of rain.

10. Naomi has a 3% chance of winning a raffle. What is the probability that Naomi will win the raffle?

11. The probability that Allanna will win the next chess game she plays is $\frac{1}{4}$. What are the chances Allanna will win the next chess game she plays?

12. The radio forecast stated that there is a 100% chance of rain on Friday and an 80% chance of rain on Saturday.

a. What is the probability of rain on Friday?
b. What is the probability of rain on Saturday?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer each of the following questions.

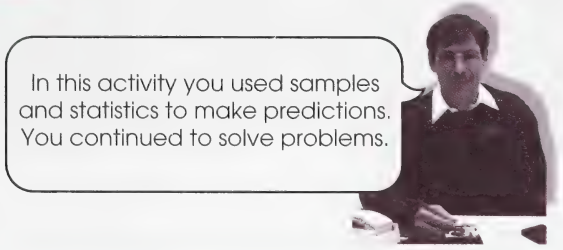
13. Using only three 9s and any mathematical symbol or operation, write an expression equivalent to 24. **Note:** Mathematical symbols and/or operations may be repeated.

14. An advertisement for a book gave the following costs:

- 1–20 books ..... $15.00 each
- 21–40 books ..... $12.50 each
- 41 or more books ..... $10.00 each

What is the minimum cost for 38 books?

Check your answers by turning to the Appendix.



Activity 3: The Monte-Carlo Method

When pilots learn to fly passenger jets, they practise in simulators before they try to fly the real planes. Pilots can experience such simulations as turbulence or engine failure without leaving the ground.

In much the same way, you can use models called **simulations** to solve complicated probability problems.

The technique where events are simulated using probability tools such as coins, dice, and dials is called the **Monte-Carlo method**.

Example 1

A football team has a win-loss-tie ratio of 5:3:2. What is the probability that the team will win or tie its next three games?

Solution

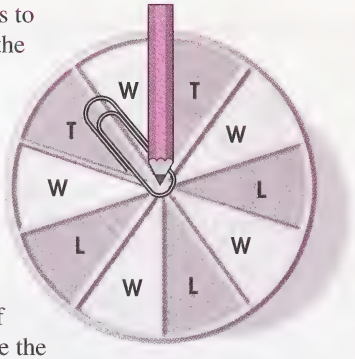
Step 1: Analyse the problem and design a model that will help you solve this problem.

What is the probability that the football team will win or tie its next three games?

The football team has a win-loss-tie ratio of 5:3:2.

$5+3+2=10$

You can use a dial like this to simulate the outcomes of the games.



You can twirl the spinner on the dial three times to represent the next three games.

Note: You can make the dial using a heavy sheet of paper or card stock. Divide the dial into 10 equal sectors. Label 5 sectors with W for win, 3 sectors with L for lose, and 2 sectors with T for tie.

You can use a sharpened pencil and a paper clip for the spinner.

Step 2: Make a table like this.

	Outcomes		
	Spin 1	Spin 2	Spin 3
Trial 1			
Trial 2			
Trial 3			
Trial 4			
Trial 5			

Note: The more trials that are conducted, the more accurate the estimate of the probability will be.

Step 3: For the first trial, twirl the spinner three times and record each result in the table. Repeat for the second trial, third trial, and so on.

Answers will vary. Here is a table of possible outcomes.

	Outcomes		
	Spin 1	Spin 2	Spin 3
Trial 1	W	W	T
Trial 2	L	W	W
Trial 3	W	T	L
Trial 4	W	W	W
Trial 5	W	L	L

Step 4: Count the number of trials with only wins and/or ties.

	Outcomes			3 Wins and/or Ties
	Spin 1	Spin 2	Spin 3	
Trial 1	W	W	T	✔
Trial 2	L	W	W	
Trial 3	W	T	L	
Trial 4	W	W	W	✔
Trial 5	W	L	L	

Answers will vary. Here is one count.

In 100 trials, there were 38 favourable outcomes.

Step 5: Calculate the relative frequency. Then use the relative frequency to estimate the probability that the team will win or tie its next three games.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{38}{100} = 0.38$$

The probability that the team will win or tie its next three games is about 0.38.

Use the Monte-Carlo method to solve each of the following problems.

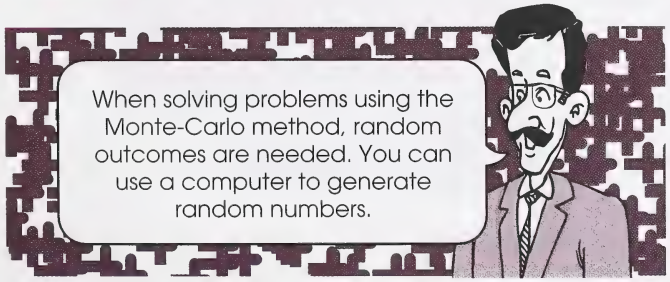
1. In the past, Jacques has won 1 out of every 6 car races in which he has competed. What is the probability that Jacques will win exactly 1 of his next 4 car races?

2. In each of 8 widely separated cities, the forecaster states that there is a 50% chance of rain on Wednesday. What is the probability that it will rain on Wednesday in 5 of the 8 cities?

3. There are 10 multiple-choice questions on a quiz. For each question there are four possible outcomes—1 correct answer and 3 wrong answers. Therefore, there is a 25% chance of getting a correct answer by guessing. If Sandra guesses the answers to the 10 multiple-choice questions, what is the probability that Sandra will get 5 or more correct answers?

4. A balloon manufacturer has determined that there is a 10% chance that a balloon will burst before it is fully inflated. What is the probability of blowing up 6 balloons in a row without bursting any?

Check your answers by turning to the Appendix.


When solving problems using the Monte-Carlo method, random outcomes are needed. You can use a computer to generate random numbers.

In the spreadsheet program in *ClarisWorks*™ there is a built-in formula that will call up random numbers. Entering the formula $=\text{RAND}(2)$ will randomly select the number 1 or 2. Entering the formula $=\text{RAND}(3)$ will randomly select the number 1, 2, or 3. Entering the formula $=\text{RAND}(4)$ will randomly select the number 1, 2, 3, or 4. Other spreadsheet programs have similar built-in formulas.

Example 2

In the past, Fred has made 4 out of every 5 free throws that he has attempted. What is the probability that Fred will make the next 2 free throws he attempts?

Solution

Methods may vary. This is one solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet program to help you solve the problem.

What is the probability that Fred will make the next 2 free throws?

In the past, Fred has made 4 out of every 5 free throws that he has attempted.

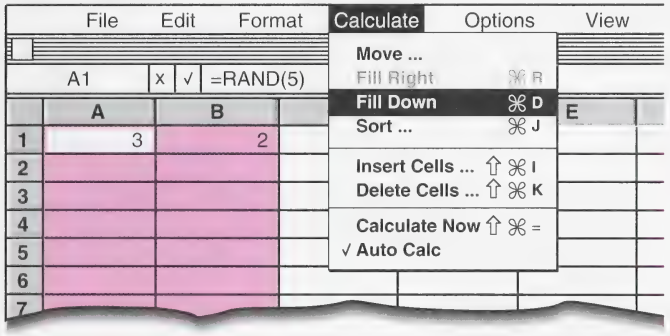
Use the formula $=\text{RAND}(5)$ to simulate each free throw. Let 1, 2, 3, and 4 represent a successful free throw. Let 5 represent an unsuccessful free throw.

Use two cells to represent the 2 free-throw attempts.

Step 2: Open a new spreadsheet file. Use row 1 to show the results of the first trial. Enter = RAND(5) in cells A1 and B1.

A random number (either 1, 2, 3, 4, or 5) will appear in each cell.

Step 3: To simulate 100 trials highlight cell A1 and shift-click on cell B100. Then click and hold on "Calculate" in the menu bar. Select "Fill Down" in the pop-up menu.



A random number (either 1, 2, 3, 4, or 5) will appear in each cell in columns A and B of the first 100 rows. Print the page.

Step 4: Count the number of rows with two successful free throws (the rows with a 1, 2, 3, or 4 in each cell).

Answers will vary. Here is an example.

In 100 trials, there are 64 favourable outcomes.

Step 5: Calculate the relative frequency. Then use the relative frequency to estimate the probability of Fred making the next two free-throw attempts.

Answers will vary. This relative frequency is based on the count in Step 4.

$$\frac{64}{100} = 0.64$$

The probability of Fred making the next two free throws is about 0.64.

Use a computer and a spreadsheet program such as *ClarisWorks*™ to answer questions 5 to 7.

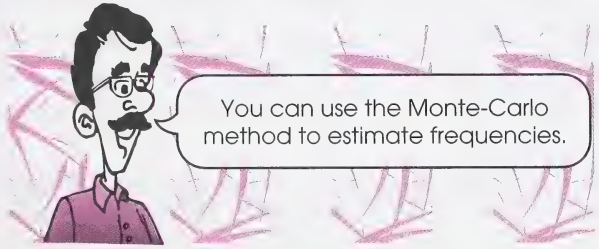
5. A two-stage rocket must fire in both stages for a successful flight. The stages fire independently. In the past, each stage fired 3 out of 4 times. What is the probability that a two-stage rocket will have a successful flight?

6. A pharmaceutical company discovers from quality control tests that certain antacid tablets fall below prescribed strength 3 out of 50 times.

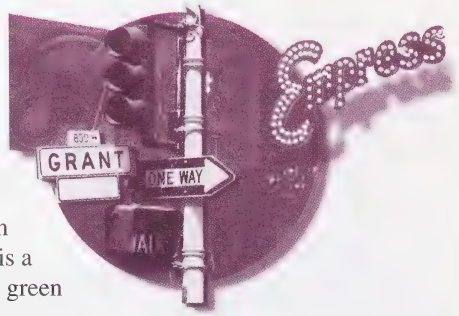
   What is the probability that there will be two or more ineffective dosages in a bottle of 20 antacid tablets?

7. Records show that in a particular city, car accidents are equally likely to occur on each day of the week. What is the probability that if two accidents occur in the city during a certain week, they will both be on the weekend (that is, Saturday and/or Sunday)?

Check your answers by turning to the Appendix.



Example 3



There are two traffic lights on Shawn's route to work. Each light has a 60-s cycle. That is, it is green for 30 s, amber for 5 s, and red for 25 s. Because each traffic light is green one-half the time, there is a 50% chance of getting a green light.

How many workdays per week would you expect Shawn to get two green lights on his way to work? **Hint:** There are 5 workdays per week. The lights operate independently of each other.

Solution

Method 1: Using Probability Tools

Methods may vary. Here is one solution.

Step 1: Analyse the problem and design a model that will help you solve the problem.

How many days per week would you expect Shawn to get two green lights on his way to work?

There are 2 traffic lights on Shawn's route to work. There is a 50% chance of getting a green light.

You can use the face of a coin to simulate the colour of a each traffic light. Let "Heads" represent a green light and "Tails" represent an amber or red light.

Use two coins to represent the two lights.

In each trial, you will need to flip the pair of coins five times (once for each of the five workdays).

Step 2: Make a table like this.

	Outcomes	
	2 "Heads"	Other
Trial 1		
Trial 2		
Trial 3		

Step 3: For Trial 1, flip a pair of coins five times and use tally marks to record how many times two "Heads" occur. (Recording how many times other combinations of "Heads" and "Tails" occur will help you ensure that there are five flips per trial.) Repeat for Trial 2, Trial 3, and so on.

	Outcomes	
	2 "Heads"	Other
Trial 1	//	///
Trial 2	/	////
Trial 3		𝍸

Step 4: Count the number of times two "Heads" occurred in a trial. Then find the total frequency in all the trials.

	Outcomes		Frequency of 2 "Heads"
	2 "Heads"	Other	
Trial 1	//	///	2
Trial 2	/	////	1
Trial 3		𝍸	0

Answers will vary. Here is one count.

In 50 trials, 2 "Heads" occurred a total of 65 times.

Step 5: Find the average number of times 2 "Heads" occurred per trial. Use this average to estimate how many workdays per week you would expect Shawn to get two green lights on his way to work.

Answers will vary. This average is based on the count in Step 4.

$$\frac{65}{50} = 1.3$$

Shawn should get two green lights on his way to work about 1.3 workdays per week.

Method 2: Using a Computer

Methods may vary. Here is one solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet program to help you solve the problem.

How many days per week would you expect Shawn to get two green lights on his way to work?

There are 2 traffic lights on Shawn's route to work. There is a 50% chance of getting a green light; that is, the probability of getting a green light is $\frac{1}{2}$.

$$50\% = \frac{50}{100}$$
$$= \frac{1}{2}$$

Use the formula $= \text{RAND}(2)$ to simulate the colour of a traffic light. Let 1 represent a green light. Let 2 represent an amber or red light.

Use a pair of cells to represent the two lights.

Use five pairs of cells to represent the five workdays.

Step 2: Use row 1 to show the results of the first trial. Enter $= \text{RAND}(2)$ in cells A1 and B1, D1 and E1, G1 and H1, J1 and K1, and M1 and N1.

A random number (either 1 or 2) will appear in each cell.

Step 3: To simulate 100 trials, highlight cell A1 and shift-click on cell N100. Then click and hold on "Calculate" in the menu bar. Select "Fill Down" in the pop-up menu.

Random numbers will appear in columns A, B, D, E, G, H, J, K, M, and N of the first 100 rows. Print the page(s).

Step 4: Count the number of times two 1s occurred in a pair of cells in each row. Then find the total frequency of a pair of 1s in all trials.

Answers will vary. Here is one count.

In 100 trials, a pair of 1s occurred 92 times.

Step 5: Find the average number of times that a pair of 1s occurred per trial. Use this average to estimate how many workdays per week you would expect Shawn to get two green lights on his way to work.

Answers will vary. This average is based on the count in Step 4.

$$\frac{92}{100} = 0.92$$

Shawn should get two green lights on his way to work about 0.92 workdays per week.

Use a computer or probability tools to answer the following questions.

8. People can be classified as extroverts, introverts, and ambiverts. Extroverts are very outgoing. Introverts are very shy. Ambiverts have a personality that is between these extremes.

It is estimated that the ratio of extroverts to introverts to ambiverts is 1:1:1 in the general population.

If a researcher forms six pairs by picking people at random, about how many of the pairs would you expect to consist of one introvert and one extrovert?

9. The entrance test for a police force consists of two parts—an academic part and an athletic-ability part. On the academic part, 5 out of 6 applicants usually pass. On the athletic-ability part, 1 in every 2 applicants usually passes. About how many of the next five applicants would you expect to pass both parts of the entrance test?

Hint: You will need two probability tools or two formulas.

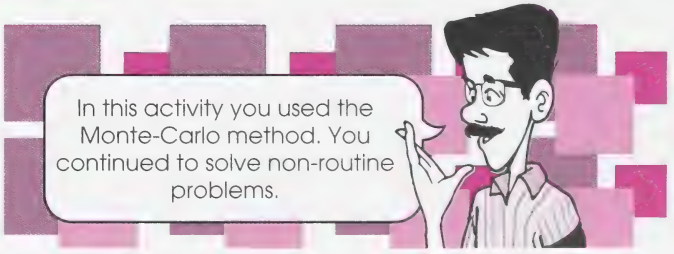
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

10. How can you place eight checkers on this checkerboard so that no checker is in the same row, column, or diagonal as any other?

Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

The law of large numbers states that the greater the number of trials of an experiment you conduct, the closer the experimental results will be to the theoretical results. However, in a small number of trials, the experimental results may be very different from the theoretical results. Therefore, sample size should be considered when you are using surveys to make predictions about a population.

1. Sharon works as a nurse in a maternity ward of a hospital. During her shift 7 babies are born. All the babies are boys.

Sharon predicts the next baby born will be a girl. Sharon reasons that the probability of a baby being a girl is $\frac{1}{2}$, so 50% of the babies should be girls. If the next baby is a girl the sample would be a more even representation of boys and girls.

Is Sharon's reasoning flawed? Why or why not?

2. The following chart shows three occasions when polls incorrectly predicted the results of elections in Canada.

'95 Ontario Election

	Environics Poll* May 12, 1995	Election Result June 8, 1995
Liberal	52%	30 seats
PC	26%	82 seats
NDP	17%	17 seats
Other	5%	1 seat

'95 Edmonton Civic Election

	Reid Poll* Oct. 6, 1995	Election Result Oct. 16, 1995
Reimer	48%	35.9%
Smith	25%	36.5%
Ramsey	15%	19.2%
Other	12%	9.4%

'93 Federal Election

	Reid Poll* Sept. 8-9, 1993	Election Result Oct. 25, 1993
Liberal	37%	177 seats
PC	35%	2 seats
Reform	10%	52 seats
BQ	8%	54 seats
NDP	8%	9 seats
Other	2%	1 seat

* All poll results are based on decided voters.

Why do you think the polls and the election results varied so greatly?

3. Pat and Julia want to know the probability of rain in their community on any day in April.

After observing the weather for 30 days one April, Pat concludes that the probability of rain on any day in April is 0.1.

Julia reviews the April weather records for her community. Using the weather data for the past 50 years, she concludes that the probability of rain on any day in April is 0.3.

Whose prediction is likely a better indication of the probability of a rainy day in April? Explain.

4. Find and complete the puzzle "When the Boy Tire Maker Married the Girl Tire Maker, What Did Everyone Say?" in the Appendix.

Check your answers by turning to the Appendix.

Enrichment

In this section you worked with probability and chance. In this Enrichment you will work with **odds**.

The odds **in favour** of an event happening compare favourable outcomes to unfavourable outcomes. The odds **against** an event happening compare unfavourable outcomes to favourable outcomes.

Example 1

The horse Morning Star has won 1 out of every 6 races in which he has competed. What are the odds against Morning Star winning his next race?

Solution

Step 1: Use this reasoning.

Morning Star has won 1 out of every 6 races. This means the horse has won 1 race for every 5 he has lost.

$$6 - 1 = 5$$

Step 2: Calculate the odds against Morning Star winning his next race.

$$\text{Odds against} = \frac{\text{unfavourable outcomes}}{\text{favourable outcomes}}$$

$$= \frac{5}{1}$$

The odds against Morning Star winning are 5 to 1.

Example 2

Maxine has 8 nickels and 6 quarters in her pocket. If she takes out one coin at random, what are the odds in favour of it being a nickel?

Solution

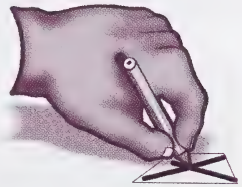
$$\text{Odds in favour} = \frac{\text{favourable outcomes}}{\text{unfavourable outcomes}}$$

$$= \frac{8}{6}$$

$$= \frac{4}{3}$$

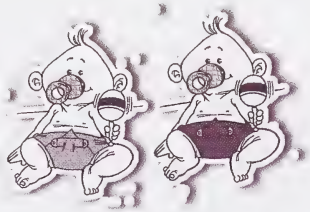
The odds in favour of the coin being a nickel are 4 to 3.

1. In Canadian federal elections, 3 out of 4 eligible voters usually cast their votes. What are the odds in favour of a person voting?



2. A jury member is chosen from a list of 17 women and 13 men. What are the odds in favour of the chosen person being a woman?

3. A woman gives birth to twins in 1 out of 96 births. What are the odds against a woman having twins?



Check your answers by turning to the Appendix.

Did You Know?

Odds are often used in discussing horse racing. Find and read the article entitled "The Web Barrier" in the Appendix. Then answer the following questions.

4. **a.** Who invented the web barrier?
b. How did the web barrier help horse racing to be more fair?

Check your answers by turning to the Appendix.

Conclusion

In this section you used games of chance to review your understanding of probability and to extend your knowledge. You used samples, statistics, and simulations to make predictions.

Statistics and probability are used in many fields from business to medicine. Governments and environmental groups use mathematics to assess the risks of various undertakings such as the construction of a nuclear-power plant, a hazardous-waste plant, or a mine. Doctors use mathematics to predict the likelihood that an operation or a treatment plan will be successful. Insurance companies use mathematics to determine which groups of drivers are most likely to have accidents.

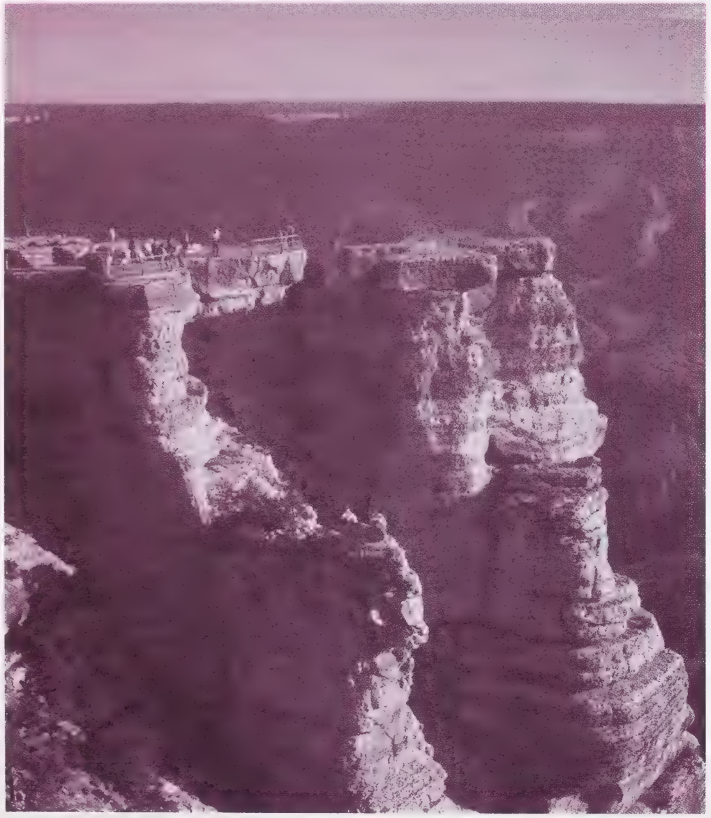
How do you use statistics and probability? Before you make a decision do you consider the likelihood of certain events happening?

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 1.

Section 2: Data Management



When people go on vacation, they usually buy postcards of the local scenery and take photographs. The postcards and photos help the tourists share their experiences with family and friends.

In statistics, as in other parts of life, a picture is worth a thousand words. Therefore, whenever it is feasible, mathematicians take the opportunity to draw pictures, or graphs, of their data.

In this section you will collect data and display the data in graphs. You will use line plots, stem and leaf plots, and box and whisker plots to analyse the distribution and central tendency of a set of data. You will use blocks to reconstruct a set of data, given various statistics.

Activity 1: Collecting Data

Scientists collect data through observation and experimentation.

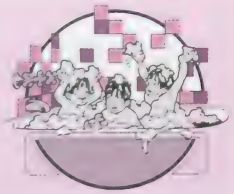
Reporters obtain data from others by asking questions and doing research.

Polling companies such as the Angus Reid Group and The Gallup Organization gather data by conducting surveys.

Data gathered directly are called **first-hand** or **primary data**. Data obtained from others are called **second-hand** or **secondary data**.

1. The following information could be obtained directly. How could you find the data?

a. the number of baths or showers that a bar of soap can be used

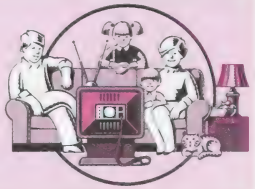


b. the number of cars that turn left every hour at a particular intersection in your community

c. the brand of toothpaste recommended by most dentists in your community

d. the amount of daily rainfall during May in your community

e. the amount of time per day that members of your family spend watching television



f. the number of people in your community that prefer one brand of cola to another brand

2. The following information would be best gathered indirectly. Where could you find the following information?

   a. the individual batting statistics for the Toronto Blue Jays

   b. the depth of the deepest lake

   c. the present population of Canada

   d. the automobile presently ranked highest for safety

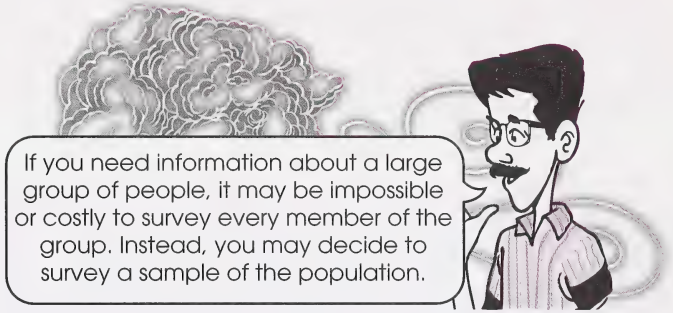
   e. the number of people who attended the opening ceremonies of the 1996 Atlanta Olympics

3. What are some advantages and disadvantages of using primary data versus secondary data?

4. There are three main methods of surveying people—interviewing them in person, interviewing them by telephone, or mailing them a questionnaire. What are some advantages and disadvantages of each method?

Check your answers by turning to the Appendix.

You may wish to use the Internet to visit the sites for the Angus Reid Group and The Gallup Organization. You may even participate in a current survey.


If you need information about a large group of people, it may be impossible or costly to survey every member of the group. Instead, you may decide to survey a sample of the population.

When you gather data directly by sampling, you need to avoid bias. Keep the following suggestions in mind:

- The sample should be as large as possible.
- The sample should be representative of the population. For example, if the target population is teenagers, then the sample surveyed should be a group of teenagers.
- The members of the sample should be chosen at random. That is, everyone in the target population should have the same chance of being chosen.
- Questions should be worded so that they do not encourage people to give a particular answer.

5. Selecting names from a hat is a common type of random selection.

List three other ways that you might randomly select a sample of 10 people from a group of 100.

6. **a.** How can bias be avoided when choosing a sample of 10 items for a quality-control test in a factory?

b. How can bias be avoided when selecting a sample of 10 people for a telephone survey?

7. The following data were obtained from a survey in which passengers on long airplane flights were asked which inflight service they considered most important.

Service	Percent
Food service	39
Flight attendant attitude/behaviour	34
Carry-on baggage compartments	10
Inflight entertainment	17

a. Who was the target population for this survey?

b. Would the results have been different if the people surveyed had only been on short airplane flights or had not travelled by air? Explain.

8. Following are three survey questions and the sample groups surveyed. List the bias that each sample might have when it responds to the question.

Question	Sample
a. Should animals be used for laboratory testing?	• scientists • animal-rights groups
b. Should the Internet be censored?	• civil-rights lawyers • parents
c. Should abortions be illegal?	• right-to-life groups • right-to-choice groups

9. What are some issues other than bias that you should consider when you are asking questions for research?

10. a. Suppose you are doing research to determine how people in Alberta feel about VLTs (video lottery terminals). Which of the following question(s) would you ask? Why?

- Since so many people are becoming addicted to gambling, do you think VLTs should be banned in Alberta communities?
- Since banning VLTs would mean that 30% of hotels would face closure, do you think VLTs should be kept in Alberta communities?
- Do you think the number of VLTs in Alberta is too low, about right, or too high?
- Are you in favour of banning VLTs in Alberta?

b. On February 24, 1997, the residents of the town of Rocky Mountain House, Alberta, voted in a **plebiscite** (a direct vote by qualified voters on a specific question) on VLTs. This is the question that was asked:

> Should the town of Rocky Mountain House request that the provincial government remove video lottery terminals from the town?

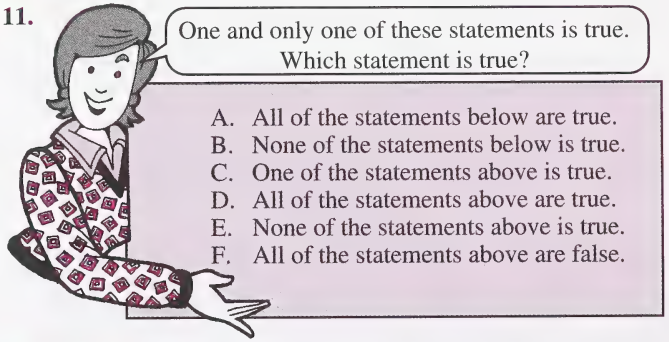
Explain why this was a "good" plebiscite question.

Check your answers by turning to the Appendix.

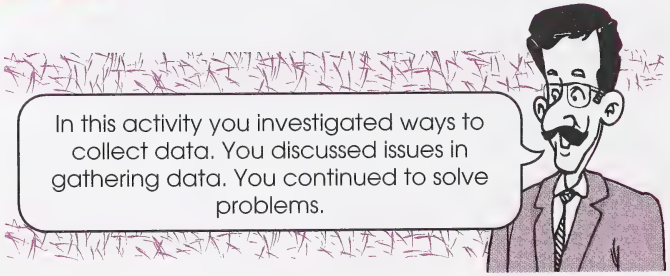
Now Try This

Use a problem-solving strategy to answer the following question.

11.


One and only one of these statements is true. Which statement is true?
A. All of the statements below are true.
B. None of the statements below is true.
C. One of the statements above is true.
D. All of the statements above are true.
E. None of the statements above is true.
F. All of the statements above are false.

Check your answer by turning to the Appendix.


In this activity you investigated ways to collect data. You discussed issues in gathering data. You continued to solve problems.

Activity 2: Analysing the Distribution of a Set of Data

The 1997 Canadian women's national figure skating championship was held in Vancouver, B.C. Susan Humphreys won the championship. Angela Derochie placed second and Jennifer Robinson placed third.

JON MURRAY, *THE PROVINCE*

The total score given by each judge to the top skaters is given in the following chart. **Note:** In figure skating, each judge gives a skater a score for technical merit (out of a possible 6.0 marks) and a score for presentation (out of a possible 6.0 marks).

Judge	Skater		
	Susan Humphreys	Angela Derochie	Jennifer Robinson
1	10.9	10.4	10.2
2	11.1	10.4	10.7
3	11.1	10.7	10.4
4	11.0	10.5	10.3
5	10.8	10.5	10.1
6	11.0	10.6	10.5
7	10.8	10.7	10.5
8	11.0	10.3	10.3
9	10.9	10.6	10.4

Three statistics that describe the distribution of each of these sets of marks are the **lower extreme**, **upper extreme**, and **range**.

The lower extreme is the least value in the set of data. The upper extreme is the greatest value in the set. The range is the difference between the upper extreme and lower extreme.

To find the lower extreme, upper extreme, and range of Jennifer Robinson's scores, arrange the data in ascending order (from least to greatest).

10.1 10.2 10.3 10.3 10.4
10.4 10.5 10.5 10.7

The lower extreme of Jennifer Robinson's scores is 10.1. The upper extreme is 10.7.

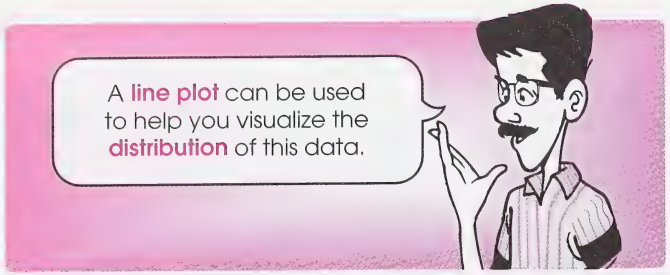
$10.7 - 10.1 = 0.6$

The range of Jennifer Robinson's scores is 0.6 marks.

1. a. What is the lower extreme of Susan Humphrey's scores? What is the upper extreme?

b. Calculate the range of Susan Humphrey's scores.

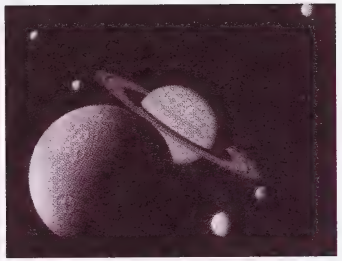
Check your answers by turning to the Appendix.



A line plot is a simple way of organizing and displaying data. A line plot is also called a **data plot**. The distribution of a set of data is the spread of the data.

Example 1

Following is a list of the planets in our solar system and the number of satellites (moons) per planet.



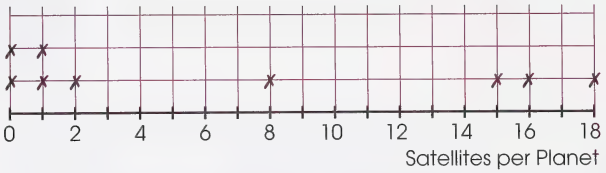
JPL

Mercury 0	Venus 0	Earth 1
Mars 2	Jupiter 16*	Saturn 18*
Uranus 15	Neptune 8	Pluto 1

*Sources disagree about the number of satellites or moons. This was the most common number given in 1997.

Use the list to make a line plot of the number of satellites per planet.

Solution



Note: A line plot shows all the values.

The lower extreme and upper extreme are easily seen on a line plot. You can use these statistics to calculate the range.

Clusters and **gaps** in the data are also visible on a line plot.

Clusters are values grouped closely together in a set of data. Gaps are spaces between values in a set of data.

Use the line plot of the number of satellites per planet in Example 1 to answer question 2.

2. **a.** What is the lower extreme of the data?
b. What is the upper extreme of the data?
c. Calculate the range of satellites per planet.
d. Are there any clusters in the data?
e. Are there any gaps in the data?

3. Alice collected the data for the June 15th high temperature in her community for the past 20 years.

Following are the temperatures (in °C) listed in chronological order (in the order they occurred).

15	17	14	14	16
19	18	13	13	13
19	17	20	20	18
15	16	21	21	21

a. Make a line plot of the data.
b. What is the lower extreme?
c. What is the upper extreme?
d. Calculate the range in temperatures.
e. Are there any clusters in the data?
f. Are there any gaps in the data?

4. Frank rolled a standard die 12 times.

Following is a list of the numbers Frank rolled, in chronological order.

3	1	3	6
2	5	4	1
2	3	1	1

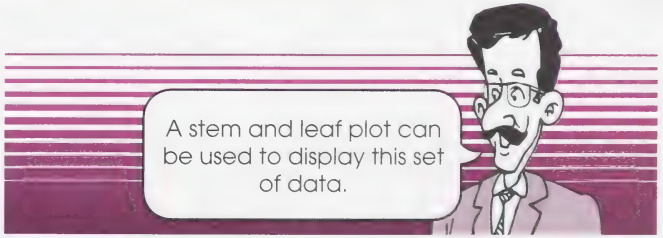
a. Make a line plot of the data.
b. What is the lower extreme?
c. What is the upper extreme?
d. Calculate the range of numbers rolled.
e. Are there any clusters in the data?
f. Are there any gaps in the data?

Check your answers by turning to the Appendix.

In 1993, Jean Chrétien became Canada's 20th prime minister.

Following is a list of the ages at which Canada's 20 prime ministers first began their terms in office. The ages are arranged in ascending order (from least to greatest).

39	45	46	46	47	48
48	51	52	54	55	57
59	60	61	65	66	70
70	74				



Here is the stem and leaf plot of the prime ministers' ages.

Stem	Leaf	
3	9	
4	5, 6, 6, 7, 8, 8	
5	1, 2, 4, 5, 7, 9	← This leaf is read as 51, 52, 54, 55, 57, 59.
6	0, 1, 5, 6	
7	0, 0, 4	

Following is another way of making a stem and leaf plot of the same data. Notice that in this stem and leaf plot, each interval is divided into two parts. The • indicates the interval is continued on a second row.

Stem	Leaf	
3		
•	9	
4		
•	5, 6, 6, 7, 8, 8	
5	1, 2, 4	← This leaf is read as 51, 52, 54.
•	5, 7, 9	← This leaf is read as 55, 57, 59.
6	0, 1	
•	5, 6	
7	0, 0, 4	

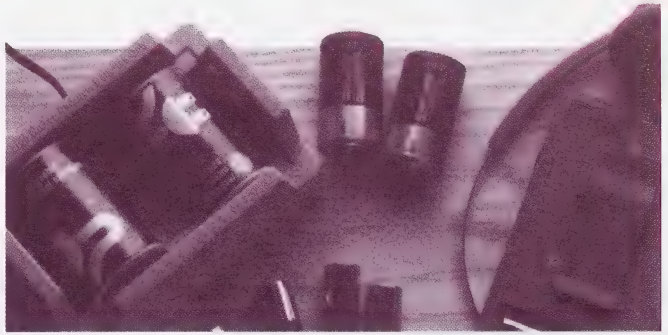
5. Use the second stem and leaf plot of the prime ministers' ages to answer the following questions.

a. What is the lower extreme of the data?
b. What is the upper extreme of the data?
c. Calculate the range of ages.
d. Are there any clusters in the data?
e. Are there any gaps in the data?

Use the Internet to find out more about Canada's prime ministers. A site you may find very interesting is the Canadian archives at the following URL:

http://cnet.unb.ca/achn/pme/

6. Following is a list of the time (in h) that each of 20 batteries lasted before needing to be recharged.



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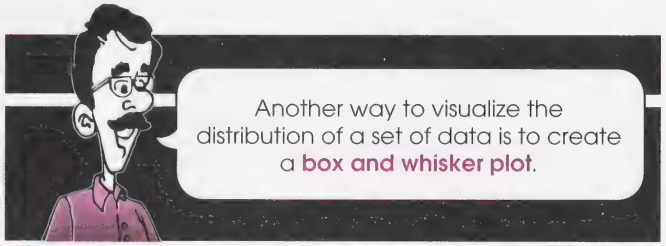
63	71	82	69	76	82	70	75	66	75
74	74	79	63	90	78	86	59	78	50

Use this data to answer the following questions.

a. Make a stem and leaf plot of the data.
b. What is the lower extreme?

c. What is the upper extreme?
d. Calculate the range of the times.
e. Are there any clusters in the data?
f. Are there any gaps in the data?

Check your answers by turning to the Appendix.



A box and whisker plot is a summary graph. It shows only certain statistics—the **upper and lower extremes**, the **median**, and the **lower** and **upper quartiles**.

When a set of data is arranged in ascending order, the middle value (or the average of the two middle values) is the median.

In the set of values less than the median, the middle value (or average of the two middle values) is the lower quartile.

In the set of values greater than the median, the middle value (or average of the two middle values) is the upper quartile.

Example 2

Over a two-week period a small neighbourhood restaurant kept a record of the number of people who sat on the patio for lunch. Following are the results.

12	11	15	7	10	9	13
9	12	13	8	11	8	12

Display the data in a box and whisker plot.

Solution

Step 1: Arrange the data in ascending order (from least to greatest).

7	8	8	9	9	10	11
11	12	12	12	13	13	15

Step 2: Find the median number of people.

There are 14 values. The median is the average of the 7th and 8th values.

7	8	8	9	9	10	11
11	12	12	12	13	13	15

← median

The median is 11 people.

Step 3: Find the lower and upper quartiles.

There are 7 values less than the median. The lower quartile is the middle value of this set.

lower quartile

7	8	8	(9)	9	10	11
11	12	12	(12)	13	13	15

← median

There are 7 values greater than the median. The upper quartile is the middle value of this set.

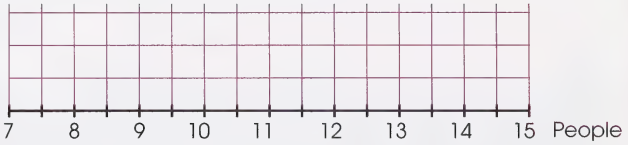
upper quartile

The lower quartile is 9 people; the upper quartile is 12 people.

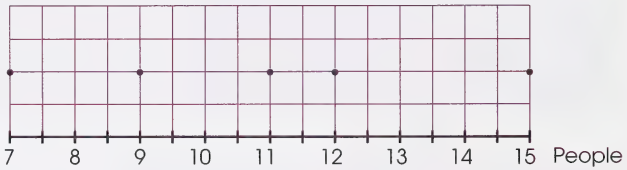
Step 4: Find the lower and upper extremes.

The lower extreme is 7 people; the upper extreme is 15 people.

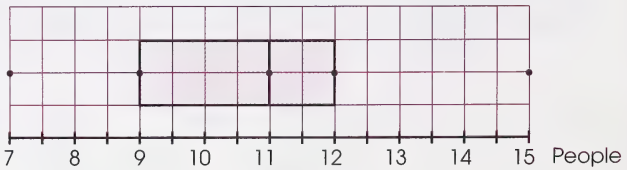
Step 5: Draw a number line that will include the lower and upper extremes. **Note:** Choose the appropriate graph paper and scale for the number line.



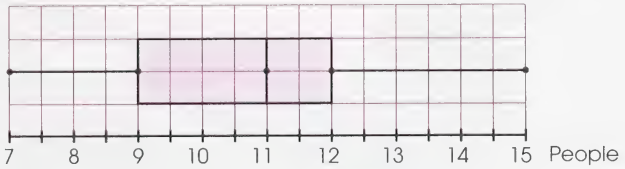
Step 6: Plot the median, lower and upper quartiles, and lower and upper extremes above the number line.



Step 7: Draw a box using the lower and upper quartiles as the ends of the box. Draw a vertical line through the box at the median.



Step 8: Draw whiskers (horizontal lines) from the ends of the box to the extremes.



Example 3

Randy is a quarterback for the Maroons.

Following is a list of the yards that he gained by passing in nine games.

89	54	67	43	54	77	58	72	88

Make a box and whisker plot of the data.

Solution

Step 1: Arrange the data in ascending order.

43	54	54	58	67	72	77	88	89

Step 2: Find the median number of yards gained.

43	54	54	58	(67)	72	77	88	89

median

The median is 67 yards.

Step 3: Find the lower and upper quartiles.

43	54	54	58	(67)	72	77	88	89

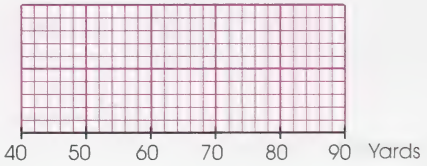
lower quartile — median — upper quartile

The lower quartile is 54 yards; the upper quartile is 82.5 yards.

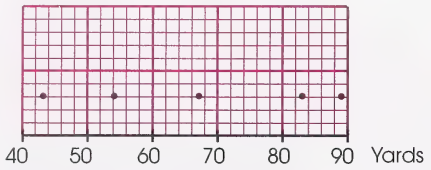
Step 4: Find the lower and upper extremes.

The lower extreme is 43 yards; the upper extreme is 89 yards.

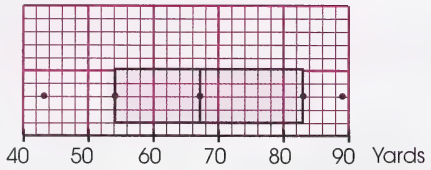
Step 5: Draw a number line that will include the extremes.
Note: Choose the appropriate graph paper and scale for the number line.



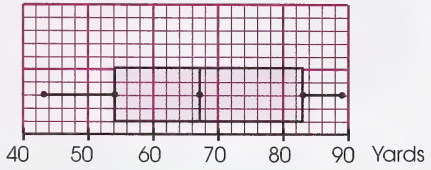
Step 6: Plot the median, lower and upper quartiles, and lower and upper extremes above the number line.



Step 7: Draw a box using the lower and upper quartiles as the ends of the box. Draw a vertical line through the box at the median.



Step 8: Draw whiskers from the ends of the box to the extremes.



In a box and whisker plot, the whiskers show where the lower quarter and upper quarter of the data are located. The ends of the whiskers indicate the upper and lower extremes. You can use the extremes to calculate the range.

In a box and whisker plot, the box shows where the middle two-quarters of the data are located. The vertical line in the box shows where the median is. The ends of the box indicate the lower and upper quartiles. You can use the quartiles to calculate the **interquartile range**.

The interquartile range is equal to the upper quartile minus the lower quartile.

7. Following is a list of the number of grams of carbohydrates per serving in each of 13 brands of breakfast cereal.

19	26	25	24	23
26	23	21	28	23
23	25	20		

a. Make a box and whisker plot of the data.
b. Calculate the range of the data.
c. Calculate the interquartile range of the data.

8. Leona volunteers for her community's reading program for the visually impaired.

Following is a list of the number of hours that Leona volunteered for the past 10 days.

5	5	6	0	3
3	4	2	5	3

a. Make a box and whisker plot of the data.
b. Calculate the range of the data.
c. Calculate the interquartile range of the data.

9. Following is a list of the average price (in $) of a first-class hotel room in each of seven cities.

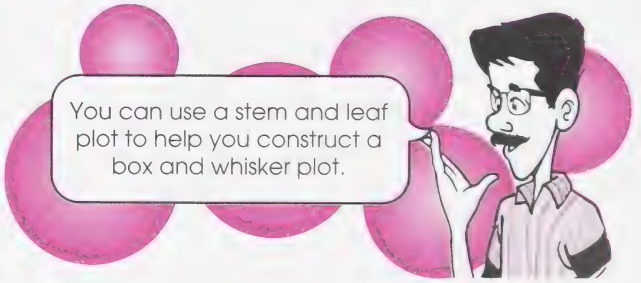
120	100	105	110	140	115	135

a. Make a box and whisker plot of the data.

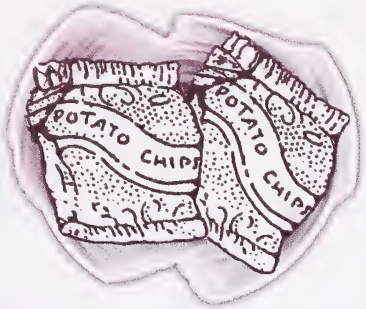
b. Calculate the range of the data.

c. Calculate the interquartile range of the data.

Check your answers by turning to the Appendix.



Example 4



A random sample of 20 bags of potato chips produced by the Tasty Tater Company was found to have different masses.

Following is the stem and leaf plot of the masses (in grams).

Stem	Leaf
6	8
7	0, 0, 2, 4, 5, 6, 6, 7, 8, 9
8	0, 0, 4, 4, 6, 8, 8
9	2, 4

Make a box and whisker plot of the data.

Solution

Step 1: Find the median mass. **Hint:** Because data are already arranged in ascending order in the stem and leaf plot, you may use the stem and leaf plot to find the median.

Stem	Leaf
6	8
7	0, 0, 2, 4, 5, 6, 6, 7, 8, | 9
8	0, 0, 4, 4, 6, 8, 8
9	2, 4

median

There are 20 values. The median is the average of the 10th and 11th values.

The median mass is 78.5 g.

Step 2: Find the lower and upper quartiles.

There are 10 values less than the median. The lower quartile is the average of the 5th and 6th values in this set.

lower quartile

Stem	Leaf
6	8
7	0, 0, 2, 4, | 5, 6, 6, 7, 8, | 9
8	0, 0, 4, 4, | 6, 8, 8
9	2, 4

upper quartile

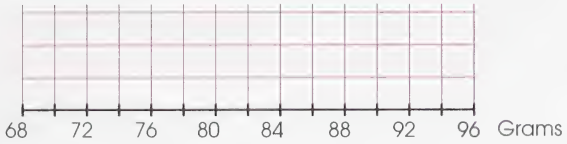
There are 10 values greater than the median. The upper quartile is the average of the 5th and 6th values in this set.

The lower quartile is 74.5 g; the upper quartile is 85 g.

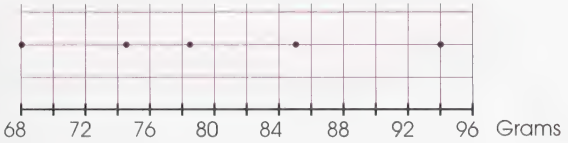
Step 3: Find the lower and upper extremes.

The lower extreme is 68 g; the upper extreme is 94 g.

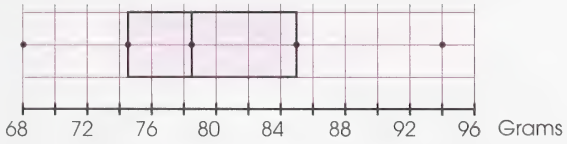
Step 4: Draw a number line that will include the extremes.



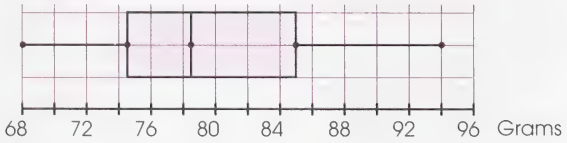
Step 5: Plot the median, lower and upper quartiles, and lower and upper extremes above the number line.



Step 6: Draw the box around the middle half of the data using the lower and upper quartiles as the ends of the box. Draw a line vertically through the box at the median.



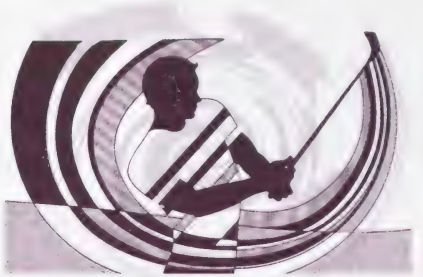
Step 7: Draw the whiskers from the box to the extremes.



10. Compare the stem and leaf plot and box and whisker plot in Example 4.

a. What are some advantages of a stem and leaf plot?
b. What are some advantages of a box and whisker plot?

11. On Saturday, 25 golfers took part in an 18-hole tournament.



Following is a stem and leaf plot which shows the scores of the 25 golfers.

Stem	Leaf
6	7, 7, 9
7	1, 2, 4, 8, 9, 9
8	4, 5, 7, 8, 9
9	2, 3, 4, 4, 6, 7, 8
10	0, 2, 4, 6

a. Make a box and whisker plot of the data.
b. Calculate the range of the data.
c. Calculate the interquartile range of the data.

12. To investigate the garbage output of small businesses in a certain community, a sample of 35 small businesses was surveyed.

Following is a stem and leaf plot which shows the number of trash cans each of the 35 businesses filled per year.

Stem	Leaf
11	0, 1, 8
12	4, 5, 9, 9
13	0, 0, 1, 4, 5, 7, 7, 7, 9, 9
14	0, 2, 2, 2, 3, 5, 5, 5, 5, 6
15	0, 2, 2, 7, 8, 9, 9, 9

a. Make a box and whisker plot of the data.
b. Calculate the range of the data.
c. Calculate the interquartile range of the data.

Check your answers by turning to the Appendix.

Did You Know?

Before the 1960s, most Canadian families put their garbage out in trash cans. Today most families use garbage bags.

Read the article entitled "Garbage Bags" in the Appendix. Then answer the following questions.

13. **a.** Who invented plastic garbage bags?
b. What is the significance of this invention?
c. Why was green chosen as the colour of garbage bags?
d. Why do you think some garbage bags are blue? clear?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer each of the following questions.

14. Mr. Barnett and four of his students went on a litter hunt. The group collected 138.5 kg of scrap metal, 568.5 kg of paper, and 675 recyclable beverage containers.

The group received an average deposit return of 15¢ per beverage container (bottle or can). The group sold the paper for 5.2¢/kg. They sold the scrap metal for $4.25/kg.

a. How much money was made by the group?

b. What was the average contribution (in dollars) by each person in the group (including Mr. Barnett)?

15. a. What is the last digit of the standard form of 4^2?

b. What is the last digit of the standard form of 4^3?

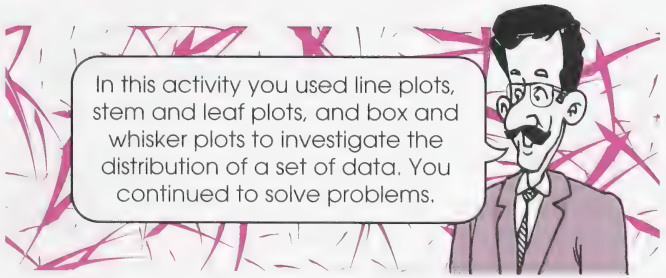
c. What is the last digit of the standard form of 4^4?

d. What is the last digit of the standard form of 4^5?

e. What is the last digit of the standard form of 4^{40}?

Hint: Do not evaluate 4^{40}.

Check your answers by turning to the Appendix.


In this activity you used line plots, stem and leaf plots, and box and whisker plots to investigate the distribution of a set of data. You continued to solve problems.

Activity 3: Analysing the Central Tendency of a Set of Data

Jessica wants to become a veterinarian some day. She wonders, "About how much does a veterinarian earn per year?"

Jason wants to buy a mountain bike. He wonders, "About how much does a decent, but reasonably priced mountain bike cost?"

Martha wants to buy an aquarium and some tropical fish. She wonders, "About how many tropical fish will a medium-size aquarium hold?"

When Jessica, Jason, and Martha ask, "About how much?" or "About how many?" they want to know the average of a set of data.

The statistic that gives the average is called a **measure of central tendency**. There are three measures of central tendency—the **median**, the **mean**, and the **mode**.

The mean is the arithmetic average of a set of data. The mode is the value that occurs most frequently in a set of data.

1. The following frequency table shows the hairstyles worn by a sample of 30 twelfth-grade females in 1970.

Haircut	Frequency
Long, straight hair	19
Afro	8
Asymmetric cut	3

The word *mode* is related to a French word meaning *fashion*. In what way can the mode of this set of numbers be thought of as "fashionable"?

2. Following is a list of the ages of the Smith's grandchildren. The data are arranged in ascending order (from least to greatest).

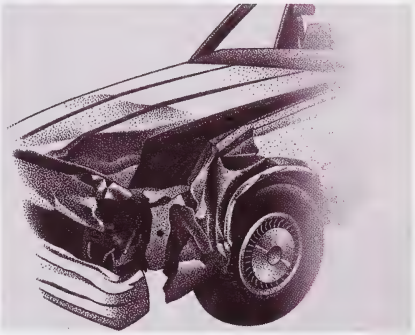
1 2 3 5 10 20

a. What is the median age of the grandchildren?

b. What is the mean age of the grandchildren?

c. What is the mode of the ages of the grandchildren?

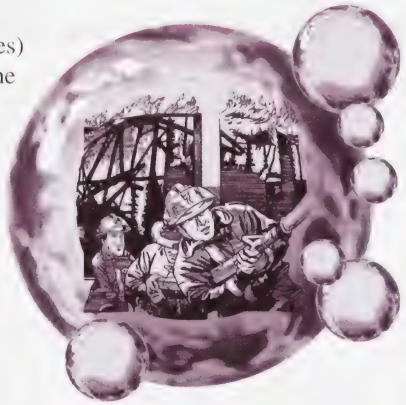
3. Following is a list of the number of accidents that occurred each day during a one-week period at a busy intersection. The data are arranged in chronological order (in the order they occurred).



3 2 0 2 5 0 2

a. What is the median number of accidents per day?
b. What is the mean number of accidents per day?
c. What is the mode of the set of data?

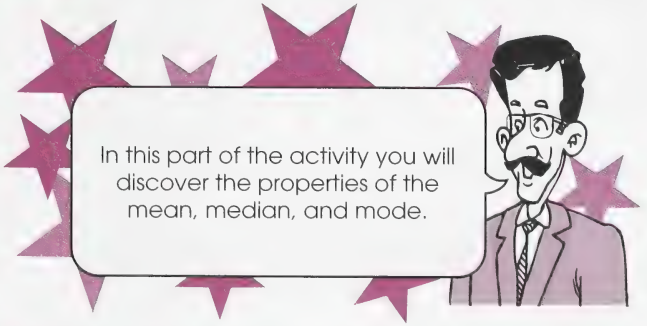
4. Following is a list of the response times (in minutes) at a fire department for the last 15 alarms. The data are arranged in chronological order.



1	3	2
2	1	9
4	6	1
10	1	4
5	10	1

a. What is the median response time?
b. What is the mean response time?
c. What is the mode of the set of data?

Check your answers by turning to the Appendix.



5. Jaydn couldn't wait to tell her friend the good news. The teacher has decided to add 3 bonus points to each student's score on today's quiz.

On today's quiz six students got the following scores before the bonus points were added.

0	1	1	3	6	7

a. What is the mode of the given scores before the bonus points are added? What is the mode after the bonus points are added?

b. What is the median score before the bonus points are added? What is the median after the bonus points are added?

c. What is the mean score before the bonus points are added? What is the mean after the bonus points are added?

d. Make a line plot of the scores before the bonus points are added. Label the mode, median, and mean.

e. Make a line plot of the scores after the bonus points are added. Label the mode, median, and mean.

f. What effect does adding the bonus points have on the mode, median, and mean?

6. The price (in $) of the same model of calculator in each of six stores is reported as follows.

14.75	15.00	15.80	15.80	16.25	17.00

Competition from other manufacturers forces the price in each store to decrease by $1.65.

a. What is the mode price before the price decrease? after the price decrease?

b. What is the median price before the price decrease? after the price decrease?

c. What is the mean price before the price decrease? after the price decrease?

d. Make a line plot of the prices before the price decrease. Label the mode, median, and mean.

e. Make a line plot of the prices after the price decrease. Label the mode, median, and mean.

f. What effect does the price decrease have on the mode, median, and mean?

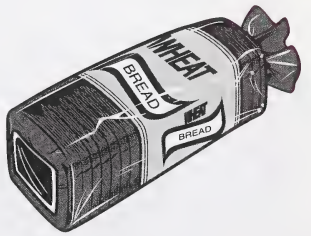
7. Following is a list of the prices (in $) of a loaf of bread in ten cities in 1970.

0.30	0.35	0.38	0.40	0.40
0.42	0.45	0.45	0.45	0.50

In 1997 the price of bread in these ten cities was 4 times the price in 1970.

a. What is the mode of the prices of bread in 1970? in 1997?

b. What is the median of the prices of bread in 1970? in 1997?



c. What is the mean of the prices of bread in 1970? in 1997?

d. Make a line plot of the prices of bread in 1970. Label the mode, mean, and median.

e. Make a line plot of the prices of bread in 1997. Label the mode, mean, and median.

f. What effect does the change in price have on the mode, mean, and median?

8. Following is a list of the yearly salaries of ten full-time employees at a grocery store.

20 000	21 000	23 000	24 000	25 000
25 000	25 000	29 000	30 000	30 000

Suppose the hours each of the employees worked were cut in half and each salary was divided by 2.

a. What is the mode of the salaries before the cut? after the cut?

b. What is the mean of the salaries before the cut? after the cut?

c. What is the median of the salaries before the cut? after the cut?

d. Make a line plot of the salaries before the cut. Label the mode, median, and mean.

e. Make a line plot of the salaries after the cut. Label the mode, median, and mean.

f. What effect does the cut have on the mode, median, and mean?

9. An investment club has five members.

Following is a list of the amount of money (in $) that each member invested at the beginning of a year.

100	500	500	650	700

At the end of the year, the members each had 120% of their original investments.

a. What is the mode value of each member's investment at the beginning of the year? at the end of the year?

b. What is the median value of each member's investment at the beginning of the year? at the end of the year?

c. What is the mean value of each member's investment at the beginning of the year? at the end of the year?

d. Make a line plot of the value of each member's investment at the beginning of the year. Label the mode, median, and mean.

e. Make a line plot of the value of each member's investment at the end of the year. Label the mode, median, and mean.

f. What effect does the increase have on the mode, median, and mean?

10. A store has a sale and advertises that all the dresses on a certain rack are now one-third off. That is, the sale price of each of the dresses on the sale rack is two-thirds of the regular price.

Following is a list of the regular prices (in $) of the 15 dresses on the sale rack.

45	48	57	57	60	60	60	66
78	78	78	78	78	84	90	

a. What is the mode price of the dresses on the sale rack before the sale? during the sale?

b. What is the median price of the dresses on the sale rack before the sale? during the sale?

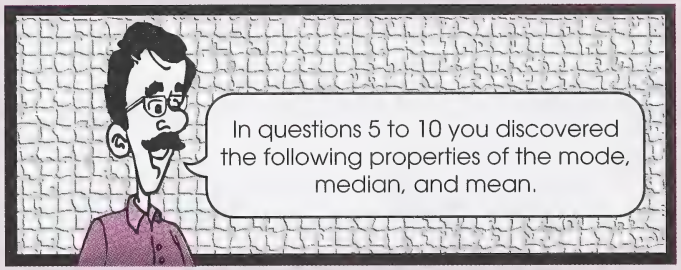
c. What is the mean price of the dresses on the sale rack before the sale? during the sale?

d. Make a line plot of the prices of dresses on the sale rack before the sale. Label the mode, median, and mean.

e. Make a line plot of the prices of the dresses on the sale rack during the sale. Label the mode, median, and mean.

f. What effect does the sale have on the mode, median, and mean?

Check your answers by turning to the Appendix.



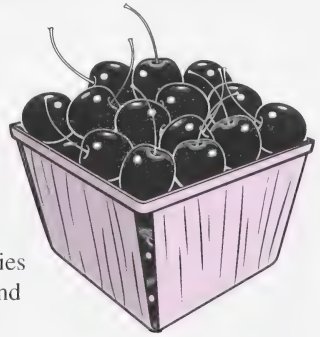
- If the same (positive) number is added to each value in a set of data, the mean, median, and mode are increased by that number.
- If the same (positive) number is subtracted from each value in a set of data, the mean, median, and mode are each decreased by that number.
- If each value in a set of data is multiplied by the same number, the mean, median, and mode are each multiplied by that number.
- If each value in a set of data is divided by the same number, the mean, median, and mode are each divided by that number.

Use these properties to answer questions 11 to 13.

11. The principal of a particular school records the number of students per classroom on the first day of school. The mean number of students per classroom is 26 and the median is 29.

If 2 extra students enrol in each classroom during the term, what is the new mean and median?

12. A survey of the price of cherries in different stores is conducted. The mean price of cherries is \$4.94/kg. The median price is \$4.89/kg.



If a heavy rain causes a shortage of cherries and the price of cherries doubles, what is the new mean and median?

13. The mean age at a family reunion is 39 and the median age is 18. If all the people at the family reunion meet again in 5 years and there are no additions to the family, what will be the new mean and median?

Check your answers by turning to the Appendix.

In questions 14 and 15 you will discover how the measures of central tendency are affected by including a significantly different value in a set of data.

14. A newspaper report gave the average salary at a small business as \$40 000 per year.

The 25 employees do not agree with this statistic.

Following is a list of the salaries of each of the 25 employees.

18 000	20 000	22 000	22 000	22 000
24 000	24 000	24 000	24 000	24 000
25 000	25 000	25 000	25 000	28 000
30 000	30 000	30 000	30 000	32 000
34 000	35 000	35 000	36 000	38 000

The owner explains that she draws a yearly salary of \$358 000 and that her salary was included in the average (mean).

a. Calculate the mean salary of the employees.
b. Calculate the median salary of the employees.

c. Calculate the median salary of the employees and owner.

d. Find the mode of the salaries of the employees.

e. Find the mode of the salaries of the employees and owner.

f. Which statistic—the mean, median, or mode—is affected the most by including the owner's salary?

g. Which statistic best represents the average salary?

15. For eight weeks last summer Warren washed and groomed dogs.

Following is a list of Warren's weekly earnings (in $).

150	175	220	280
300	300	300	325

After subtracting start-up costs of $300, Warren's weekly earnings (in $) became the following.

-150	175	220	280	300	300	300	325

Notice that at the end of his first week, Warren owed $150.

a. What is the mode of Warren's weekly earnings before subtracting the start-up costs? after subtracting the start-up costs?

b. What is the median of the weekly earnings before subtracting the start-up costs? after subtracting the start-up costs?

c. What are the mean weekly earnings before subtracting the start-up costs? after subtracting the start-up costs?

d. Which statistic—the mean, median, or mode—is affected the most by subtracting the start-up costs?

e. Which statistic best represents Warren's average weekly earnings?

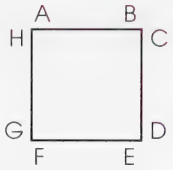
Check your answers by turning to the Appendix.

Including a significantly different value in a set of data will affect the mean more than the median or the mode. Therefore, the median or the mode usually describes the central tendency of a set of data more accurately.

Now Try This

Use a problem-solving strategy to answer the following question.

16. Andrew and his wife Anita gave a dinner party to which they invited Barbara and her husband Bob, Charlie and his wife Chloe, and Della and her husband Dave.



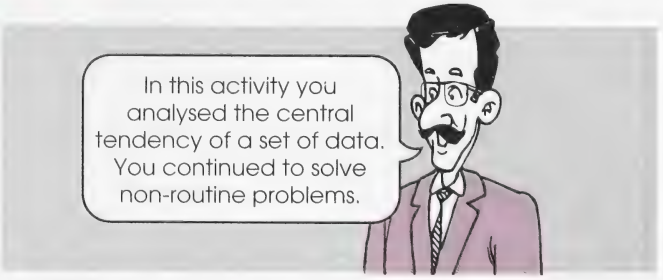
While they were all seated at the table, a glass of water was spilled on the person in chair F.

Where did each person sit? Who was the victim?

Use these clues:

- The host sat on the victim's right.
- Bob sat next to the hostess.
- Dave sat beside the victim's spouse.
- Each man sat across from his wife.
- Each man sat between a man and a woman.

Check your answers by turning to the Appendix.



Activity 4: Regaining Data

A university report states that the mean value of the scholarships given to ten students is \$2500. How much money was awarded for the ten scholarships?

To calculate the amount of money awarded for scholarships, let s represent the sum of all the scholarships and write an equation. Then solve the equation.

$$\text{mean} = \frac{\text{sum of all the scholarships}}{\text{number of scholarships}}$$

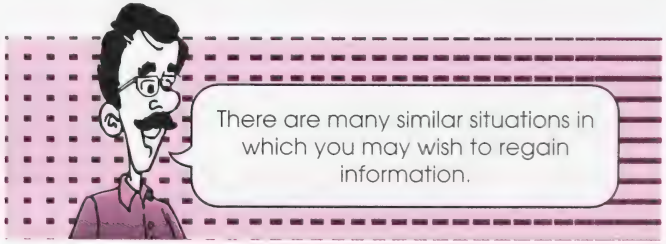
$$2500 = \frac{s}{10}$$

$$10(2500) = 10\left(\frac{s}{10}\right)$$

Multiply each side of the equation by 10.

$$25\,000 = s$$

The amount of money awarded for the ten scholarships was \$25 000.

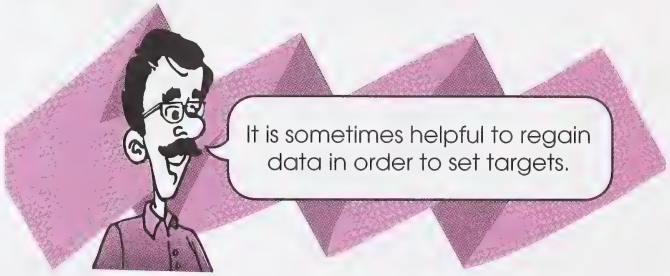


1. Frank is a barber. He works 20 days in July. Frank's mean number of haircuts per day is 12. What is the total number of haircuts that Frank does in July?

2. A server reported that she served a total of 110 tables last week and the mean tip per table is $2. How much did the server make in tips last week?

3. The mean hourly wage at a dry cleaners is $6. If there are 30 full-time employees, how much in total does the owner pay in wages per hour?

Check your answers by turning to the Appendix.



Example 1

Jason has bowled two games and has one game left. His average (mean) score in the two games is 110. What score does Jason need on the third game to increase his average to 115?

Solution

Step 1: Calculate the total points scored in the two games.

Let t represent the total points scored.

$$\text{mean} = \frac{\text{total number of points scored}}{\text{number of games}}$$

$$110 = \frac{t}{2}$$

$$2(110) = 2\left(\frac{t}{2}\right)$$

$$220 = t$$

Jason scored a total of 220 points in two games.

Step 2: Calculate the total number of points Jason must score in the three games to get a mean of 115.

Let t represent the total points.

$$\text{mean} = \frac{\text{total number of points scored}}{\text{number of games}}$$

$$115 = \frac{t}{3}$$

$$3(115) = 3\left(\frac{t}{3}\right)$$

Multiply each side of the equation by 3.

$$345 = t$$

Jason will need to score a total of 345 points in the three games.

Step 3: Calculate the score Jason must get in the third game.

$$345 - 220 = 125$$

Jason must score 125 points in the third game.

4. So far, Maury has gotten a mean of 78% on five tests this term. His target is to have a mean of at least 80%. What is the least percent Maury must get on his next test to raise his average to 80%?

5. Janice is the sales manager in a store which is open six days a week (the store is closed on Sundays). She wants to maintain an average (mean) daily sales of at least $8500.

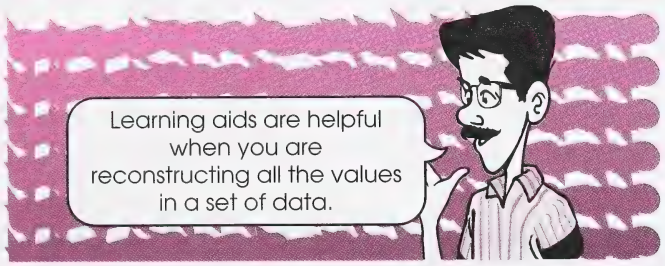
Following is a list of the daily Janice's sales (in dollars) for the first four days of the week.

7530	8475	6550	7155

a. What minimum sales will Janice need to make on Friday and Saturday to meet her target?

b. Do you think that Janice is likely to achieve her target? Why or why not?

Check your answers by turning to the Appendix.



In the Appendix, find the page of base ten blocks. Photocopy the page and glue it to heavy paper. Then cut out the blocks.

Alternatively, you may use wooden or plastic base ten blocks, sugar cubes, or building blocks.

Example 2

Thelma buys 5 items. The price of the least expensive item is \$1. The price of the most expensive item is \$20. The median price of the items is \$3. The mean price is \$6.

Reconstruct a set of data that has these statistics. **Hint:** Use only whole numbers.

Solution

Answers will vary. Here is one solution.

Step 1: Calculate the total price of the 5 items if the mean price is \$6.

Let t represent the total price.

$$\text{mean} = \frac{\text{total price}}{\text{number of items}}$$

$$6 = \frac{t}{5}$$

$$5(6) = 5\left(\frac{t}{5}\right)$$ Multiply each side by 5.

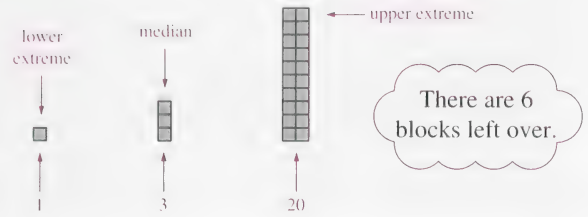
$$30 = t$$

The total price of the 5 items is \$30.

Step 2: Gather blocks to represent the total price of the 5 items.

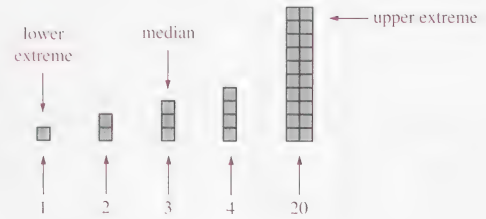
You will need 30 blocks.

Step 3: Use some of the 30 blocks to represent 3 of the 5 items. Keep in mind the median price is \$3, the lower extreme is \$1, and the upper extreme is \$20.



Step 4: Model the remaining 2 items using the remaining 6 blocks. Keep in mind that the median is \$3.

Answers may vary. Here is one solution.



A set of data that has the given statistics is \$1, \$2, \$3, \$4, and \$20.

Example 3

There were 10 people in a pie-eating contest. The least number of pie pieces any person ate is 4 and the most is 15. The mean is 9 pieces and the median is 9 pieces. The mode is 10.

Reconstruct a set of data that has these statistics.

Solution

Answers will vary. Here is one solution.

Step 1: Calculate the total number of pie pieces eaten by the 10 people.

Let t represent the total number of pie pieces eaten.

$$\text{mean} = \frac{\text{total of pie pieces eaten}}{\text{number of people}}$$

$$9 = \frac{t}{10}$$

$$10(9) = 10\left(\frac{t}{10}\right)$$

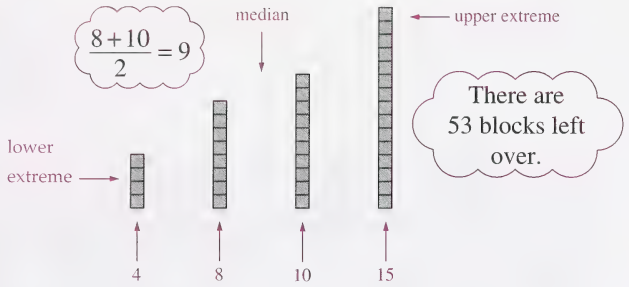
$$90 = t$$

The total number of pie pieces eaten was 90.

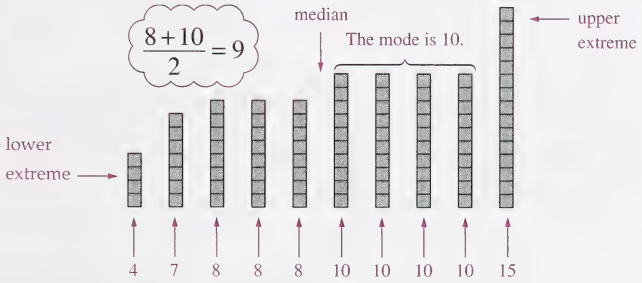
Step 2: Gather blocks to represent the number of pie pieces eaten.

You will need 90 blocks to represent the pie pieces eaten.

Step 3: Use some of the 90 blocks to represent the pieces of pies eaten by 4 of the 10 people in the contest. Keep in mind that the median is 9, the lower extreme is 4, and the upper extreme is 15.



Step 4: Use the remaining 53 blocks to represent the pieces of pies eaten by the remaining 6 people. Keep in mind the median is 9 and the mode is 10.



A set of data that has the given statistics is 4, 7, 8, 8, 8, 10, 10, 10, 10, 15.

6. **a.** Give another solution to Example 2.
b. Give another solution to Example 3.

7. Frances plays soccer. In each of the last 6 games, the least number of goals her team scored was 1 and the greatest number was 6.

The mean number of goals scored per game is 3 and the median is also 3.

Reconstruct a set of data that has these statistics.

8. A group of 13 students take a quiz. The mean score on the quiz is 5. The median is also 5, but the mode is 6. The scores range from 2 to 10.

Reconstruct a set of data that has these statistics.

9. In one hour a shoe store sells 8 pairs of shoes. The median shoe size sold is $8\frac{1}{2}$. The mean shoe size sold is 9. The mode is size 8. The smallest size of shoe sold is 7 and the largest size of shoe is 12.

Reconstruct a set of data that has these statistics. **Hint:** Use only whole numbers.

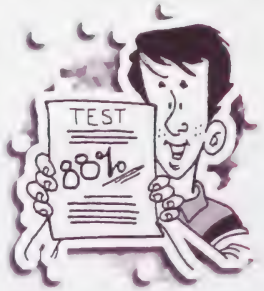
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer each of the following questions.

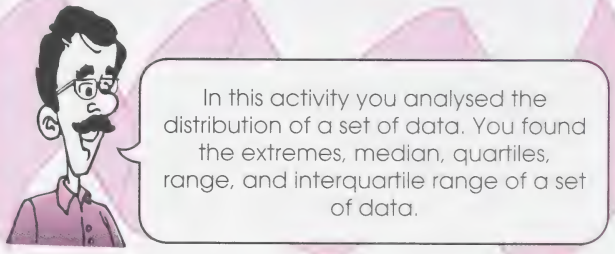
10. The mean price of 5 items from a grocery store is \$4.40. The mean price of another 10 items from the same store is \$9.80. What is the mean price of all 15 items?

11. Mark had an average of 88% on five tests.



What is the lowest percentage that Mark could have received on one test?

Check your answer by turning to the Appendix.


In this activity you analysed the distribution of a set of data. You found the extremes, median, quartiles, range, and interquartile range of a set of data.

Activity 5: Displaying Data

The weather is something that Canadians spend a lot of time thinking about.

Following is a table of the average high and low summer temperatures (rounded to the nearest °C) in several Canadian cities.

Location	Temperature °C	
	Summer	
	High	Low
Vancouver	22	13
Whitehorse	21	8
Yellowknife	21	12
Calgary	23	9
Saskatoon	25	12
Winnipeg	26	13
London	26	14
Ottawa	26	15
Toronto	27	14
Montreal	26	16
Quebec	25	13
Fredericton	26	13
Halifax	23	13
Charlottetown	23	14
St. John's	20	11

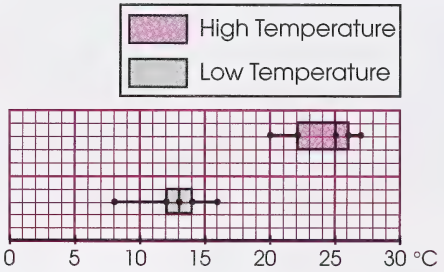
The high and low temperatures of the given cities can be displayed as a double stem and leaf plot and as a double box and whisker plot.

Here is the double stem and leaf plot of the two sets of temperatures.

High Temperature		Low Temperature
	0	
	•	8, 9
	1	1, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4 ← This leaf is read as 11, 12, 12,…
	•	5, 6
3, 3, 3, 2, 1, 1, 0	2	
7, 6, 6, 6, 6, 6, 5, 5	•	

This leaf is read as 25, 25, 26,…

Here is the double box and whisker plot of the two sets of temperatures.



These two graphs allow you to quickly compare the two sets of data.

1. a. What is the advantage of displaying the data in a double stem and leaf plot?

b. What is the advantage of displaying the data in a double box and whisker plot?

2. Queen Elizabeth II is the monarch of Britain and the other Commonwealth countries.

Following is a table that gives the name of each of the previous 15 monarchs, as well as the age at which each ascended the throne and the age at which each died.

Monarch	Age at Accession	Age at Death
Charles II	30	55
James II	51	67
William III	39	51
Mary II	27	32
Anne	37	49
George I	54	67
George II	44	77
George III	22	81
George IV	58	67
William IV	65	71
Victoria	18	81
Edward VII	60	68
George V	45	70
Edward VIII	42	77
George VI	41	57

a. Make a double stem and leaf plot of the data.
b. Make a double box and whisker plot of the data.
c. Which set of data has the greater range?
d. Which set of data has the greater interquartile range?
e. Which set of data has the greater median age?

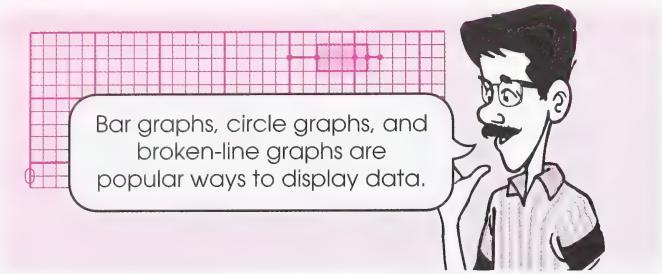
3. Each year the American Academy of Motion Picture Arts and Sciences presents Oscars for the best actor and the best actress.

Following is a table of winners (and their ages) for the years 1980 to 1997.

Best Actress	Age	Best Actor	Age
Sissy Spacek	30	Robert De Niro	37
Katharine Hepburn	74	Henry Fonda	76
Meryl Streep	33	Ben Kingsley	39
Shirley MacLaine	49	Robert Duvall	55
Sally Field	38	F. Murray Abraham	45
Geraldine Page	61	William Hurt	35
Marlee Matlin	21	Paul Newman	61
Cher	41	Michael Douglas	33
Jodie Foster	26	Dustin Hoffman	51
Jessica Tandy	80	Daniel Day-Lewis	32
Kathy Bates	42	Jeremy Irons	42
Jodie Foster	29	Anthony Hopkins	54
Emma Thompson	33	Al Pacino	52
Holly Hunter	36	Tom Hanks	37
Jessica Lange	46	Tom Hanks	38

a. Make a double stem and leaf plot of the data.
b. Make a double box and whisker plot of the data.
c. Which set of data has the greater range?
d. Which set of data has the greater interquartile range?
e. Which set of data has the greater median age?

Check your answers by turning to the Appendix.



4. In 1996, one community had a total of 3600 crimes in the categories of homicide, assault, break and enter, and car theft.

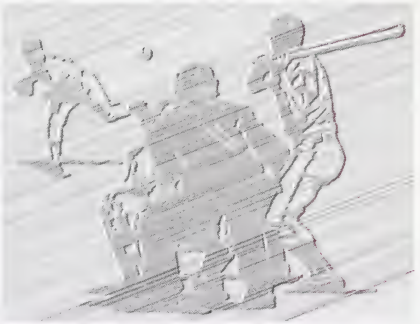
The following table shows the break-down of the crimes.

Crime	Frequency
Homicide	40
Assault	390
Break and Enter (Houses)	1670
Car Theft	1500

a. Make a bar graph of this data.
b. Make a circle graph of this data.

5. Find the puzzle entitled "What Do You Call a Person Who Buys and Sells Bugs?" in the Appendix. Photocopy the page and complete the puzzle.

6. The following table shows the minimum salaries, in U.S. dollars, for major league baseball players in different years.



Year	Minimum Salary (U.S. $)
1970	12 000
1974	15 000
1978	21 000
1982	33 500
1986	60 000
1990	100 000
1994	109 000

Make a broken-line graph of this data.

Check your answers by turning to the Appendix.

The invention of computer spreadsheets has made it easy to make and modify bar graphs, circle graphs, and broken-line graphs.

Use a computer and a spreadsheet program such as *ClarisWorks*™ to answer questions 7 to 9.

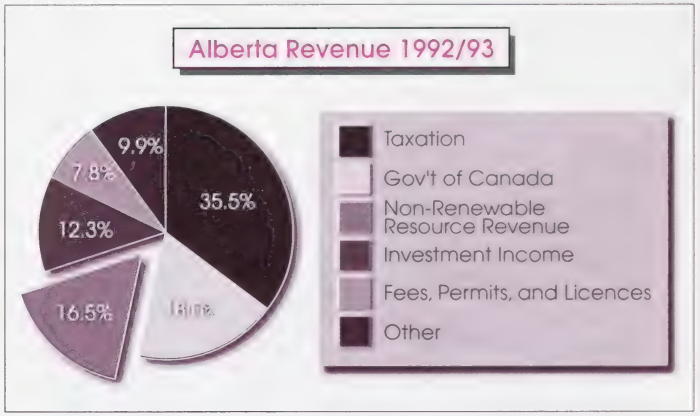
7. The government of Alberta obtains revenue from several sources.

The following table shows the sources and the amounts of money Alberta received in 1992/93.

Alberta Revenue 1992/93	
Source	**Amount ($)**
Taxation	4 695 000 000
Gov't of Canada	2 384 000 000
Non-Renewable Resource Revenue	2 183 000 000
Investment Income	1 621 000 000
Fees, Permits, and Licences	1 027 000 000
Other	1 314 000 000

Sarah made a graph of this data using *ClarisWorks*™.

Sarah's graph looked like this.



Start a new spreadsheet file. Enter the data from the table. Experiment with the "Make Chart" and "Modify Chart" options to make the computer draw a graph like Sarah's. Then answer the following questions.

a. What headings and data did Sarah enter in her spreadsheet to make the circle graph?

b. How did Sarah make and modify the circle graph?

8. The following chart groups the provinces and territories of Canada into three regions.

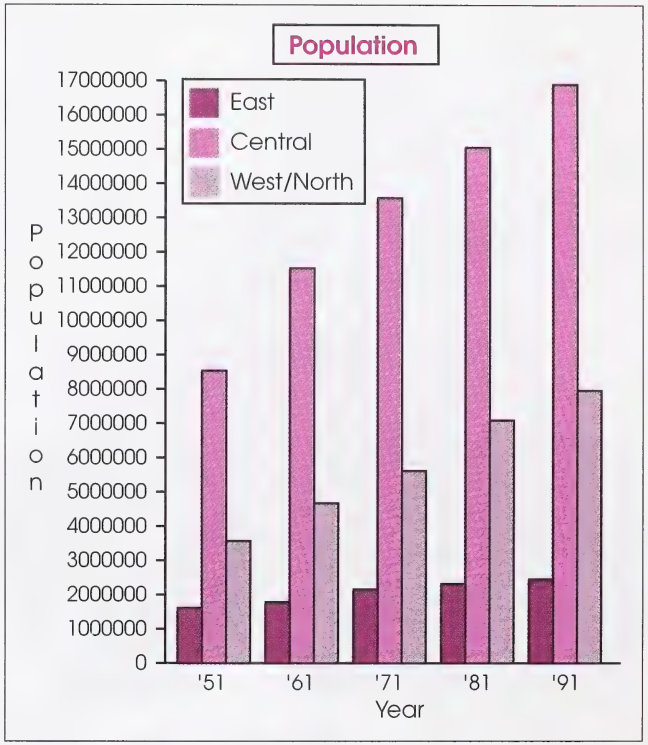
Grouping by Region		
East	**Central**	**West/North**
New Brunswick Newfoundland/ Labrador Nova Scotia Prince Edward Island	Ontario Quebec	Alberta British Columbia Manitoba Saskatchewan Yukon and Northwest Territories

The following table gives the population of each region in 1951, 1961, 1971, 1981, and 1991.

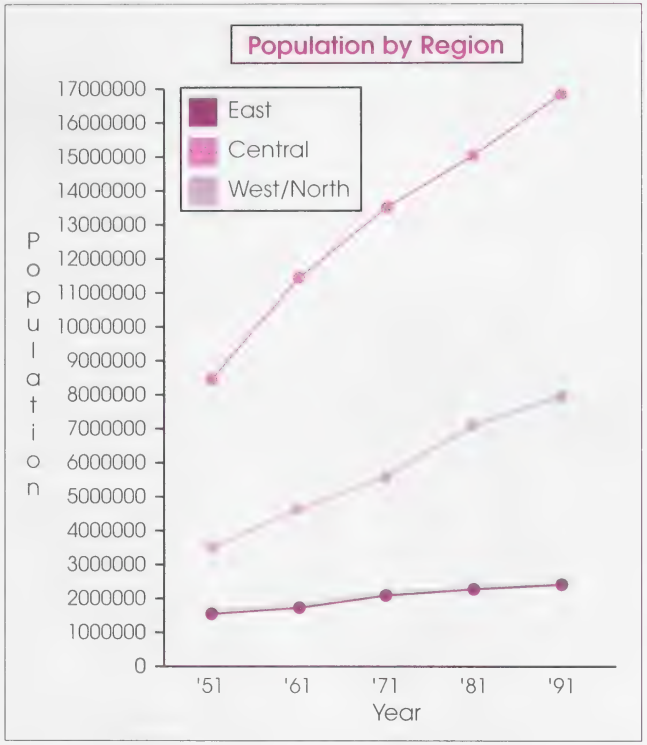
Population by Region			
Year	**Region**		
	East	**Central**	**West/North**
'51	1 618 126	8 653 223	3 738 080
'61	1 897 425	11 495 303	4 845 519
'71	2 057 264	13 730 871	5 780 185
'81	2 234 035	15 063 512	7 045 640
'91	2 322 081	16 980 848	7 990 930

To display the given data, Michael made two graphs using *ClarisWorks*™.

The first graph was a bar graph. It looked like this.



The second graph was a broken-line graph. It looked like this.



Start a new spreadsheet file. Enter the data from the table. Experiment with the "Make Chart" and "Modify Chart" options to draw computer-drawn graphs like Michael's. Then answer the following questions.

a. What headings and data did Michael enter in his spreadsheet to make the graphs?

b. How did Michael modify the bar graph?

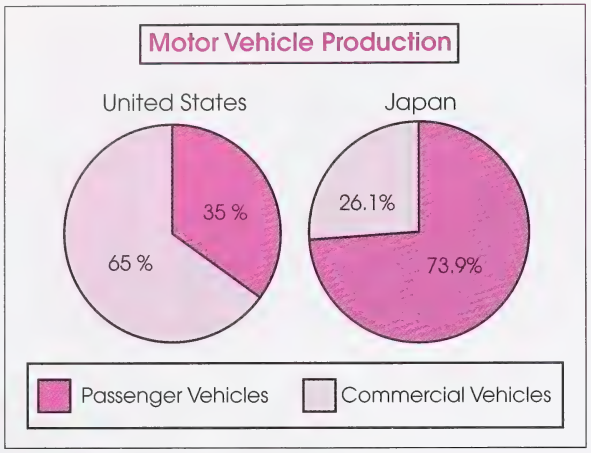
c. How did Michael modify the broken-line graph?

9. In 1994, the two countries that produced the most motor vehicles were United States and Japan. (Canada was fifth.)

The following table shows the number of passenger and commercial vehicles produced in each country that year.

Motor Vehicle Production		
Country	**Passenger Vehicles**	**Commercial Vehicles**
United States	6 613 970	12 262 737
Japan	7 801 317	2 752 802

Rosemary made the following graph to display the data.



Start a new spreadsheet file. Enter the data from the table. Experiment with the "Make Chart" and "Modify Chart" options to make a computer-drawn graph like Rosemary's. Then answer the following questions.

a. What headings and data did Rosemary enter in her spreadsheet to make the graph?

b. How did Rosemary modify the pie chart?

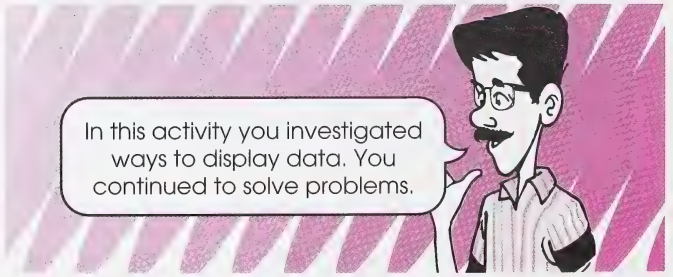
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

10. A watch uniformly loses 8 min every 24 h. If the watch was correctly set at noon on June 1, what was the time indicated by this watch at 18:00 on June 6?

Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

The mean is an important statistic.

Example 1

Following is a list of the cost (in $) of a dog licence in 9 communities.

2	2	2
4	5	5
7	8	10

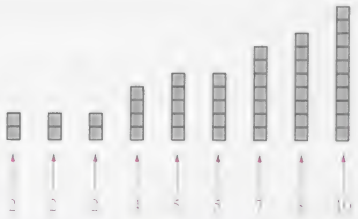
Calculate the mean cost of a licence.

Solution

$$\frac{3\times 2+4+2\times 5+7+8+10}{9}=\frac{45}{5}$$

The mean price of a dog licence is $5.

Note: You may better understand Example 1 by using blocks. Let one block represent $1 and model the given values.



If you distribute the blocks evenly, you discover that the mean cost is $5.

Example 2

In a psychology experiment each of 10 dogs learns a new trick.

The least number of trials a dog takes to learn the new trick is 5.
The most number of trials is 24.
The mean number of trials is 12.
The mode is 11 and the median is also 11.

Reconstruct a set of data that has these statistics.

Solution

Step 1: Calculate the total time for the 10 dogs to learn the new tricks.

Let t represent the total time.

$$\text{mean} = \frac{\text{total time}}{\text{number of dogs}}$$

$$12 = \frac{t}{10}$$

$$10(12) = 10\left(\frac{t}{10}\right)$$

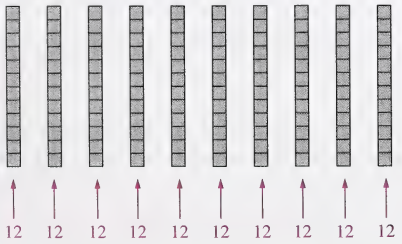
$$120 = t$$

The total time is 120 minutes.

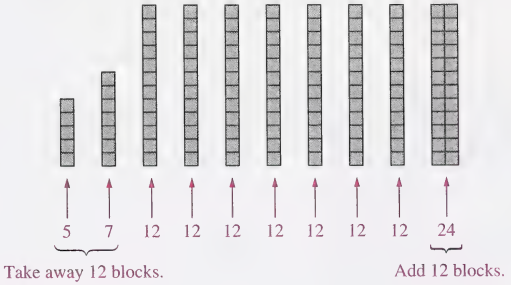
Step 2: Gather blocks to represent the total time.

You will need 120 blocks.

Step 3: The mean number of trials is 12. Therefore, arrange the blocks in 10 stacks of 12.

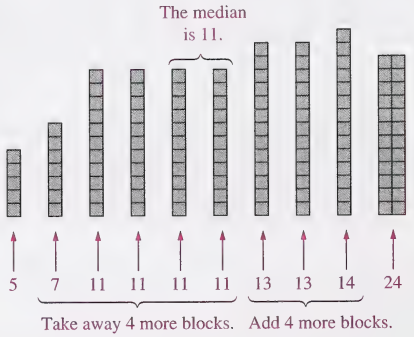


Step 4: The least number of trials is 5 and the most is 24. Therefore, make the adjustments keeping in mind that balance must be maintained.



Step 5: The mode is 11 and the median is 11. Therefore, make the adjustments keeping in mind that balance must be maintained.

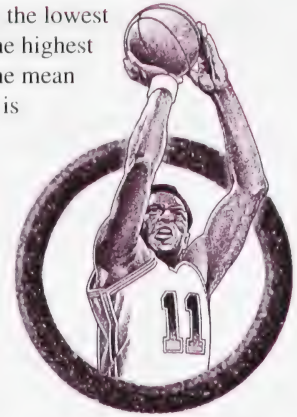
Answers will vary. Here is one solution.



A set of data that has the given statistics is 5, 7, 11, 11, 11, 11, 13, 13, 14, 24.

1. Randy plays basketball. In 6 games, the lowest number of baskets he scores is 1. The highest number of baskets he scores is 9. The mean number of baskets is 5. The median is 4.5 baskets and the mode is 9.

   Reconstruct a set of data that has these statistics.



2. A scientist records the earth vibrations per second produced by 10 waterfalls.

   The least number of vibrations per second is 3. The greatest number is 40. The mean number of vibrations is 16. The median number of vibrations is 10. The mode is 6.

   Reconstruct a set of data that has these statistics.

Check your answers by turning to the Appendix.

Enrichment

In this section you made box and whisker plots.

In this Enrichment you will explore an alternate way to make box and whisker plots for a set of data that has **outliers**.

An outlier is a value more than 1.5 times the interquartile range above the upper quartile or below the lower quartile.

Example

Fifteen students take a quiz.

Following is a list of the students' quiz scores.

7	7	5	10	9	8	7	6
8	8	7	9	8	1	2	

Make a box and whisker plot of the data.

Solution

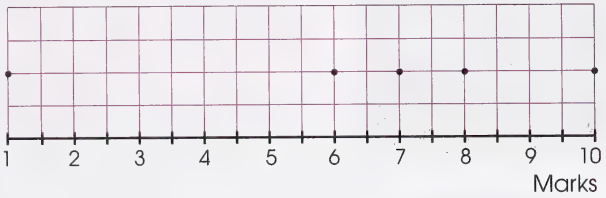
Method 1: Making a Standard Box and Whisker Plot

Step 1: Arrange the data in ascending order and find the median, quartiles, and extremes.

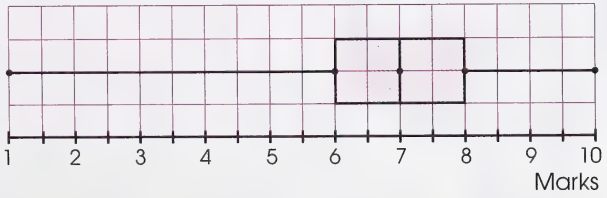
1, 2, 5, (6), 7, 7, 7, (7), 8, 8, 8, (8), 9, 9, 10

lower extreme; lower quartile; median; upper quartile; upper extreme

Step 2: Make a number line and plot the median, quartiles, and extremes.



Step 3: Draw the box using the quartiles as the ends to the box. Draw a vertical line through the box at the median. Draw the whiskers.



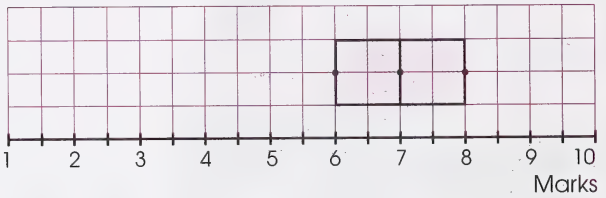
Method 2: Showing the Outliers

Step 1: Arrange the data in ascending order and find the median, quartiles, and extremes.

1, 2, 5, (6), 7, 7, 7, (7), 8, 8, 8, (8), 9, 9, 10

lower extreme; lower quartile; median; upper quartile; upper extreme

Step 2: Make a number line and plot the median and quartiles. Then make a box using the quartiles as the ends to the box. Draw a vertical line through the box at the median.



Step 3: Calculate the interquartile range and identify the outliers.

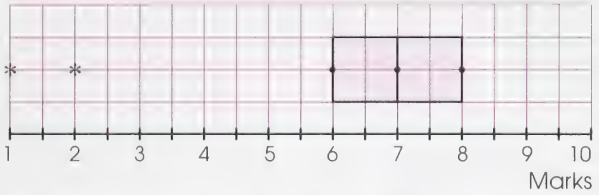
Because $8 - 6 = 2$, the interquartile range is 2.

Because $1.5 \times 2 = 3$, scores more than 3 marks below the lower quartile or more than 3 marks above the upper quartile will be outliers.

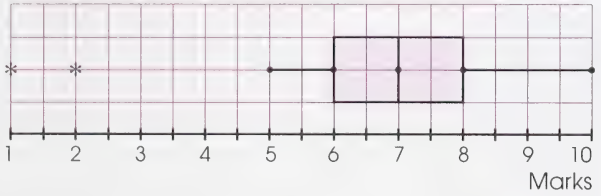
$6 - 3 = 3;\ 8 + 3 = 11$

There are two outliers—the scores of 1 and 2.

Step 4: Use an asterisk (*) to indicate each outlier on the box and whisker plot that you began in Step 2.



Step 5: Use a dot to plot the upper extreme; use a dot to plot the lowest value, not including the outliers. Then draw the whiskers. **Hint:** Outliers are not included in the whiskers.



Use the example to answer question 1.

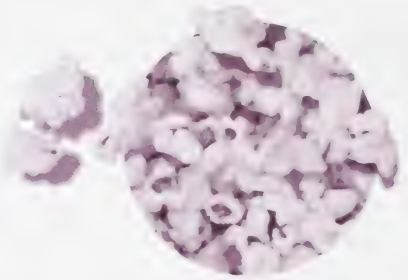
1. a. Estimate the fraction of the quiz scores between 6 (the lower quartile) and 8 (the upper quartile).

b. Estimate the fraction of the quiz scores 8 or above.

c. Estimate the fraction of the quiz scores 6 or below.

d. What is the advantage of using asterisks to show the outliers?

2. Following is a stem and leaf plot of the prices (in $) of 20 models of popcorn poppers.



Stem	Leaf
1	5, 6, 8
2	0, 1, 1, 2, 4, 5, 5, 6, 8
3	2, 4, 4, 4
4	1, 2, 5
5	9

a. What is the median price?

b. What is the lower quartile? upper quartile?

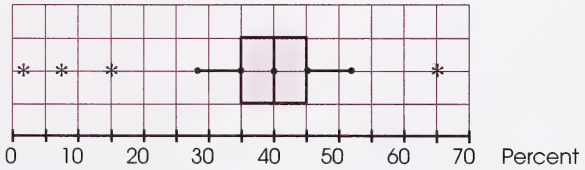
c. What is the lower extreme? upper extreme?

d. Are there any outliers? If so, what are they?

e. Make a box and whisker plot using an asterisk (*) to show each outlier.

3. Brands of cereal that children like seem to contain a high percentage of sugar.

This box and whisker plot shows the percentage of sugar in 30 kinds of cereals that children like.



a. What is the lower extreme? upper extreme?
b. Calculate the range.
c. What is the median?
d. What is the lower quartile? upper quartile?
e. Calculate the interquartile range.
f. How many outliers are there?
g. What do the outliers indicate?

4. Following is a table of the low temperatures (in °C) at noon one September 21 in each Canadian province and territory.

Province/Territory	Low Temperature (°C)
Newfoundland	3
Nova Scotia	22
New Brunswick	5
Prince Edward Island	15
Quebec	−2
Ontario	0
Manitoba	−5
Saskatchewan	−4
Alberta	−4
British Columbia	−9
Yukon	−25
Northwest Territories	−30

a. What is the median temperature?

b. What is the lower quartile? upper quartile?

c. What is the lower extreme? upper extreme?

d. Are there any outliers? If so, what are they?

e. Make a box and whisker plot using an asterisk (*) to show each outlier.

Check your answers by turning to the Appendix.

Conclusion

In this section you investigated how data are collected. You used line plots, stem and leaf plots, and box and whisker plots to analyse the distribution and central tendency of a set of data. You regained data, given various statistics. You displayed data by hand and by computer in a variety of ways.

In statistics, as in other parts of life, a picture is worth a thousand words. Therefore, whenever it is feasible, mathematicians take the opportunity to draw pictures, or graphs, of their data.

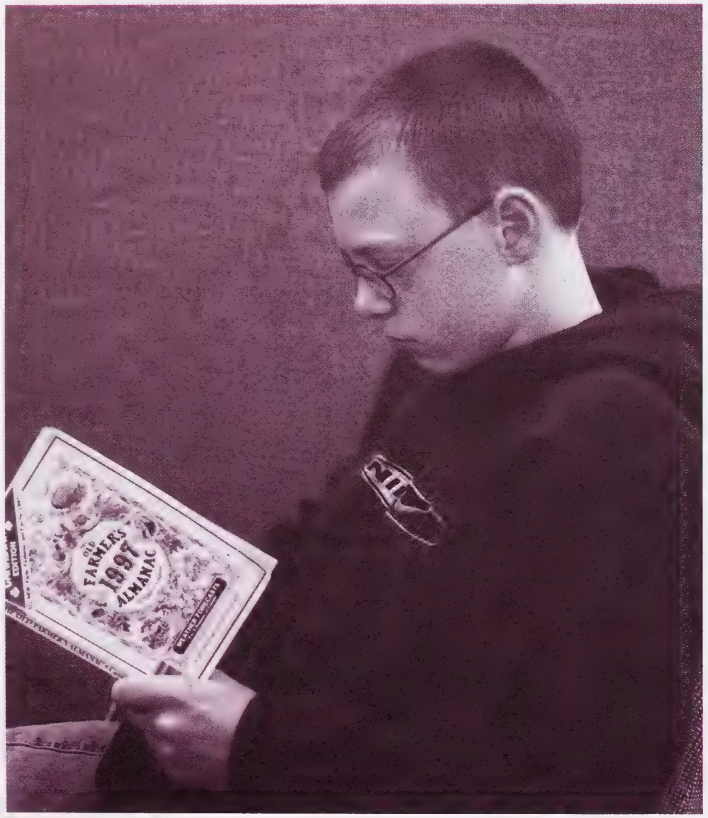
You will encounter graphs at presentations; in newspapers, magazines, and newsletters; and on television.

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 2.

Module Summary



In this module you investigated chance and uncertainty. You also explored data management.

The authors of *The Old Farmer's Almanac* use data and patterns to make yearly predictions about the weather in different parts of North America.

Do you think the predictions in *The Old Farmer's Almanac* are less accurate than those of a weather forecaster? Are the predictions in *The Old Farmer's Almanac* more accurate than folklore indicators such as the saying that if March comes in like a lion, it will go out like a lamb?

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX

Glossary

Suggested Answers

Articles/Puzzles

Glossary

Box and whisker plot: a graph that illustrates the spread of data

Capture-recapture: a sampling method used to estimate the number of wild animals in a region

Central tendency (measure of): the statistic that gives an average—median, mean, mode

Chance: the probability of an event happening, usually expressed as a percent

Clusters: values grouped closely together in a set of data

Distribution (measures of): the statistics that give the spread—extremes, range, quartiles, interquartile range, clusters, and gaps

First-hand (primary) data: data gathered directly

Gaps: spaces between values in a set of data

Interquartile range: the difference between the upper quartile and lower quartile

Line plot: a graph used to organize and display data; also called a **data plot**

Lower extreme: the least value in the set of data

Lower quartile: the middle value (or average of the two middle values) in the set of values less than the median

Mean: the arithmetic average of a set of data

Median: the middle value (or the average of the middle two values) in a set of data arranged in ascending order

Mode: the value that occurs most frequently in a set of data

Monte-Carlo method: a technique where events are modelled using probability tools such as coins, dice, and dials, or using a computer to generate random numbers

These models are called **simulations**.

Odds (against): a comparison of the unfavourable outcomes to the favourable outcomes

Odds (in favour): a comparison of the favourable outcomes to the unfavourable outcomes

Outliers: values in a set of data that are more than 1.5 times the interquartile range above the upper quartile or below the lower quartile

Population: in statistics, the set of people, animals, or things being studied

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Range: the difference between the upper extreme and lower extreme in a set of data

Relative frequency: the ratio of the number of times something has occurred to the number of times it could occur

Sample: the part of the population that is surveyed or tested

Second-hand (secondary) data: data obtained from others

Technology: the application of tools, materials, and processes to problem solving; more specifically, devices and systems used in processing, transferring, storing, and communicating information through electronic media

Theoretical probability (of an event): the ratio of the number of favourable outcomes (what you want to happen) to the number of possible outcomes (what could happen)

Upper extreme: the greatest value in a set of data

Upper quartile: the middle value (or average of the two middle values) in the set of values greater than the median

Suggested Answers

Section 1: Activity 1

1. a. The most frequently used vowel is E.
b. The two least frequently used vowels are U and Y.
c. The three most frequently used consonants are T, N, and R.
d. The three least frequently used consonants are Z, Q, and J.

2. Yes, the relative frequency of letters in the English language was probably used in determining the number of tiles with each letter. There are more tiles with E; there are fewer tiles with U and Y. There are more tiles with T, N, and R; there are fewer tiles with Z, Q, and J.

3. a. Answers will vary.

b. Generally, contestants guess the most frequently used letters more often than the less frequently used letters.

c. Answers will vary. Sometimes the final message has less frequently used letters.

4. a. **Step 1:** Find the number of favourable outcomes.

There is 1 favourable outcome.

Step 2: Find the number of possible outcomes.

There are 52 possible outcomes.

Step 3: Calculate the theoretical probability of drawing the ace of spades.

$$P(\textit{ace of spades}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$= \frac{1}{52}$$

The probability of drawing the ace of spades is $\frac{1}{52}$.

b. **Step 1:** Find the number of favourable outcomes.

There are 4 favourable outcomes—queen of clubs, queen of diamonds, queen of hearts, and queen of spades.

Step 2: Find the number of possible outcomes.

There are 52 possible outcomes.

Step 3: Calculate the theoretical probability of drawing a queen.

$$P(\textit{queen}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{4}{52}$$
$$= \frac{1}{13}$$

The probability of drawing a queen is $\frac{1}{13}$.

c. **Step 1:** Find the number of favourable outcomes.

There are 13 favourable outcomes.

Step 2: Find the number of possible outcomes.

There are 52 possible outcomes.

Step 3: Calculate the probability of drawing a diamond.

$$P(\textit{diamond}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{13}{52}$$
$$= \frac{1}{4}$$

The probability of drawing a diamond is $\frac{1}{4}$.

d. **Step 1:** Find the number of favourable outcomes.

There are 26 favourable outcomes—all the clubs and spades.

Step 2: Find the number of possible outcomes.

There are 52 possible outcomes.

Step 3: Calculate the probability of drawing a black card.

$$P(\textit{black}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{26}{52}$$
$$= \frac{1}{2}$$

The probability of drawing a black card is $\frac{1}{2}$.

e. **Step 1:** Find the number of favourable outcomes.

There are 12 favourable outcomes—3 face cards in each suit.

Step 2: Find the number of possible outcomes.

There are 52 possible outcomes.

Step 3: Calculate the probability of drawing a face card.

$$P(\textit{face card}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{12}{52}$$
$$= \frac{3}{13}$$

The probability of drawing a face card is $\frac{3}{13}$.

f. **Step 1:** Find the number of favourable outcomes.

There are 24 favourable outcomes—the 7, 6, 5, 4, 3, and 2 of each suit.

Step 2: Find the number of possible outcomes.

There are 52 possible outcomes.

Step 3: Calculate the probability.

$$P(\textit{a card lower than } 8) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{24}{52}$$
$$= \frac{6}{13}$$

The probability of drawing a card lower than eight is $\frac{6}{13}$.

g. **Step 1:** Find the number of favourable outcomes.

There are no cards higher than an ace.

Step 2: Find the number of possible outcomes.

$$P(\textit{a card higher than an ace}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{0}{52}$$
$$= 0$$

The probability of drawing a card higher than an ace is 0.

5. a. **Step 1:** Find the number of favourable outcomes.

There is 1 favourable outcome—an L.

Step 2: Find the number of possible outcomes.

There are 20 possible outcomes.

Step 3: Calculate the theoretical probability of rolling the letter L.

$$P(L) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{1}{20}$$

The probability of rolling the letter L is $\frac{1}{20}$.

b. **Step 1:** Find the number of favourable outcomes.

There are 0 favourable outcomes. (Q is not present on any face of the die.)

Step 2: Find the number of possible outcomes.

There are 20 possible outcomes.

Step 3: Calculate the theoretical probability of rolling the letter Q.

$$P(Q) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{0}{20}$$
$$= 0$$

The probability of rolling the letter Q is 0.

c. **Step 1:** Find the number of favourable outcomes.

There are 4 favourable outcomes—A, E, I, O. (The U and Y are not present on any face of the die.)

Step 2: Find the number of possible outcomes.

There are 20 possible outcomes.

Step 3: Calculate the theoretical probability of rolling a vowel.

$$P(\textit{vowel}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$
$$= \frac{4}{20}$$
$$= \frac{1}{5}$$

The probability of rolling a vowel is $\frac{1}{5}$.

6. a.

Sums		First Spin									
		10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
Second Spin	**10**	20	30	40	50	60	70	80	90	100	100
	20	30	40	50	60	70	80	90	100	over	100
	30	40	50	60	70	80	90	100	over	over	100
	40	50	60	70	80	90	100	over	over	over	100
	50	60	70	80	90	100	over	over	over	over	100
	60	70	80	90	100	over	over	over	over	over	100
	70	80	90	100	over	over	over	over	over	over	100
	80	90	100	over	over	over	over	over	over	over	100
	90	100	over	over	over	over	over	over	over	over	100
	100	over	over	over	over	over	over	over	over	over	100

b. **Step 1:** Find the number of favourable outcomes.

There are 19 favourable outcomes—10 favourable outcomes in a single spin and 9 in a combination of spins.

Step 2: Find the number of possible outcomes.

There are 100 possible outcomes.

Step 3: Calculate the probability of spinning 100 in one or two spins.

$$P(100) = \frac{\text{favourable}}{\text{possible}}$$

$$= \frac{19}{100}$$

The probability of spinning 100 in one or two spins is $\frac{19}{100}$.

c. **Step 1:** Find the number of favourable outcomes.

There are 45 favourable outcomes.

Step 2: Find the number of possible outcomes.

There are 100 possible outcomes.

Step 3: Calculate the probability of the contestant going over 100.

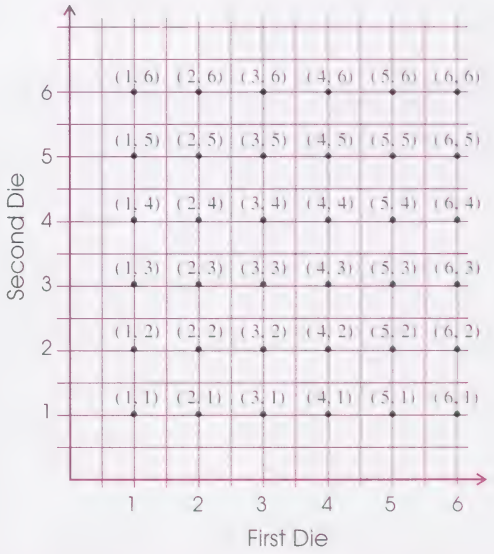
$$P(over\ 100) = \frac{\text{favourable}}{\text{possible}}$$

$$= \frac{45}{100}$$

$$= \frac{9}{20}$$

The probability of going over 100 is $\frac{9}{20}$.

7. **a.** The possible outcomes of rolling a pair of dice are shown in the following graph.



b. **Step 1:** Find the number of favourable outcomes.

There are 6 favourable outcomes—$(1,1)$, $(2,2)$, $(3,3)$, $(4,4)$, $(5,5)$, $(6,6)$.

Step 2: Find the number of possible outcomes.

There are 36 possible outcomes.

Step 3: Calculate the probability of rolling "doubles."

$$P(\textit{doubles}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{6}{36}$$
$$= \frac{1}{6}$$

The probability of rolling "doubles" is $\frac{1}{6}$.

8. **a.** Answers will vary. Generally, the game appears to be fair with neither player having an unfair advantage.

b.

Winner		Player B's Action		
		Rock	Scissors	Paper
Player A's Action	Rock	tie	Player A wins.	Player B wins.
	Scissors	Player B wins.	tie	Player A wins.
	Paper	Player A wins.	Player B wins.	tie

c. **Step 1:** Find the number of favourable outcomes.

There are 3 favourable outcomes.

Step 2: Find the number of possible outcomes.

There are 9 possible outcomes.

Step 3: Calculate the probability of Player A winning.

$$P(\textit{Player A winning}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}$$

The probability of Player A winning is $\frac{1}{3}$.

d. **Step 1:** Find the number of favourable outcomes.

There are 3 favourable outcomes.

Step 2: Find the number of possible outcomes.

There are 9 possible outcomes.

Step 3: Calculate the probability of Player B winning.

$$P(\textit{Player B winning}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}$$

The probability of Player B winning is $\frac{1}{3}$.

e. **Step 1:** Find the number of favourable outcomes.

There are 3 favourable outcomes.

Step 2: Find the number of possible outcomes.

There are 9 possible outcomes.

Step 3: Calculate the probability of a tie.

$$P(\textit{tie}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{3}{9}$$
$$= \frac{1}{3}$$

The probability of a tie is $\frac{1}{3}$.

f. Based on the theoretical probability, the game is fair. Player A and Player B have the same probability of winning.

9. **a.** Answers will vary. Generally, the game appears to be unfair with Player B having an advantage.

b.

Fractions		Second Die					
		1	2	3	4	5	6
First Die	1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
	2	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$
	3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$
	4	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$
	5	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{5}{6}$
	6	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$

c.

Player Who Scores a Point		Player B					
		1	2	3	4	5	6
Player A	1	B	B	B	B	B	B
	2	B	A	B	A	B	A
	3	B	B	A	B	B	A
	4	B	A	B	A	B	A
	5	B	B	B	B	A	B
	6	B	A	A	A	B	A

Note: The fractions $\frac{2}{2}$, $\frac{2}{4}$, $\frac{2}{6}$, $\frac{3}{3}$, $\frac{3}{6}$, $\frac{2}{4}$, $\frac{4}{4}$, $\frac{4}{6}$, $\frac{5}{5}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, and $\frac{6}{6}$ are **not** in simplist form.

d. **Step 1:** Find the number of favourable outcomes.

There are 13 ways Player A can score a point.

Step 2: Find the number of possible outcomes.

There are 36 possible outcomes.

Step 3: Calculate the probability that Player A will score a point.

$$P(\textit{Player A scoring}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{13}{36}$$

The probability of Player A scoring a point is $\frac{13}{36}$.

e. **Step 1:** Find the number of favourable outcomes.

There are 23 ways Player B can score a point.

Step 2: Find the number of possible outcomes.

There are 36 possible outcomes.

Step 3: Calculate the probability of Player B scoring a point in a round.

$$P(\textit{Player B scoring}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{23}{36}$$

The probability of Player B scoring a point is $\frac{23}{36}$.

f. **Step 1:** Find the number of favourable outcomes.

There is no way that there will be a tie.

Step 2: Find the number of possible outcomes.

The number of possible outcomes is 36.

Step 3: Calculate the probability of a tie.

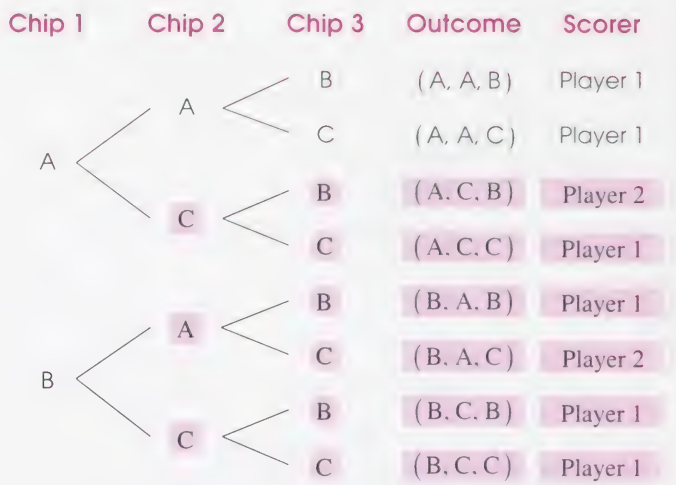
$$P(tie) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{0}{36}$$
$$= 0$$

The probability that neither Player A nor Player B will score is 0.

g. No, the game is not fair. There is a greater probability that Player B will score a point in each round.

10. a. Answers will vary. Generally, the game appears to be unfair with Player 1 having an advantage.

b. The tree should look like this.



c. **Step 1:** Find the number of favourable outcomes.

There are 6 ways Player 1 can score a point in a round.

Step 2: Find the number of possible outcomes.

There are 8 possible outcomes.

Step 3: Calculate the probability that Player 1 scores a point in a round.

$$P(\textit{Player 1 scoring}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{6}{8}$$
$$= \frac{3}{4}$$

The probability that Player 1 scores a point is $\frac{3}{4}$.

d. **Step 1:** Find the number of favourable outcomes.

There are 2 ways Player 2 can score a point in the round.

Step 2: Find the number of possible outcomes.

There are 8 possible outcomes.

Step 3: Calculate the probability that Player 2 will score a point in a round.

$$P(\textit{Player 2 scoring}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{2}{8}$$
$$= \frac{1}{4}$$

The probability that Player 2 scores a point is $\frac{1}{4}$.

e. **Step 1:** Find the number of favourable outcomes.

There is no way that there is a tie.

Step 2: Find the number of possible outcomes.

There are 8 possible outcomes.

Step 3: Calculate the probability of a tie.

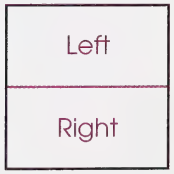
$$P(\textit{tie}) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{0}{8}$$
$$= 0$$

The probability that there is a tie is 0.

f. No, based on the theoretical probabilities, the game is not fair. Player 1 has an advantage.

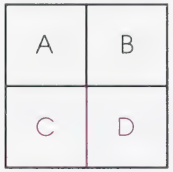
11. **Step 1:** Draw a square to help you visualize the problem.

Step 2: At the first branch in the network, the chip is equally likely to go into the left path or the right path. Therefore, divide the square into two equal parts and label the parts "Left" and "Right."



Step 3: Suppose that at the first branch the chip goes into the left path. This path splits into two more paths. One leads to A and the other leads to B. The chip is equally likely to go into either path. Therefore, divide the region for the left path into two equal parts and label the parts "A" and "B."

Step 4: Suppose that at the first branch the chip goes into the right path. This path splits into two more paths. One leads to C and the other leads to D. The chip is equally likely to go into either path. Therefore, divide the region for the right path into two equal parts and label the parts "C" and "D."



a. Use the diagram in Step 4 to calculate the probability of the chip landing at A.

The diagram shows that there is 1 favourable outcome out of 4.

$$P(A) = \frac{\text{favourable}}{\text{possible}}$$

$$= \frac{1}{4}$$

The probability of the chip landing at A is $\frac{1}{4}$.

b. Use the diagram in Step 4 to calculate the probability of the chip landing at B.

$$P(B) = \frac{\text{favourable}}{\text{possible}}$$

$$= \frac{1}{4}$$

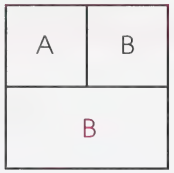
The probability of the chip landing at B is $\frac{1}{4}$.

12. Step 1: Draw a square to help you visualize the problem.

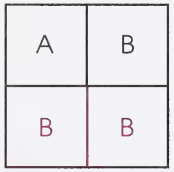
Step 2: At the first branch in the network, the chip is equally likely to go into the left path or the right path. Therefore, divide the square into two equal parts and label the parts "Left" and Right."

Step 3: Suppose that at the first branch, the chip goes left. The path splits into two more parts. One path leads to A and the other leads to B. The chip is equally likely to go into either of these paths. Therefore divide the region for the left path into two equal parts and label the parts "A" and "B."

Step 4: Suppose that at the first branch, the chip goes to the right. This path leads to B. Therefore label the right path as "B."



Step 5: The last region labelled B has twice the area of the other parts (see Step 4). To make the comparison easier, divide the region into two equal parts and label each part "B."



a. Use the diagram in Step 5 to calculate the probability that the chip will land at A.

$$P(A) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{1}{4}$$

The probability of the chip landing at A is $\frac{1}{4}$.

b. Use the diagram in Step 5 to calculate the probability that the chip will land at B.

$$P(B) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{3}{4}$$

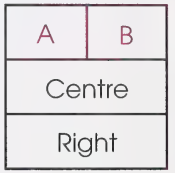
The probability is $\frac{3}{4}$.

Now Try This

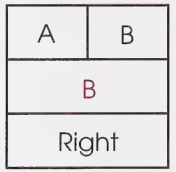
13. Step 1: Draw a square like this to help you visualize the problem.

Step 2: At the first branch in the maze, there are three doors. One door is on the left, one in the centre, and one on the right. Reynaldo is equally likely to open any of these three doors. Therefore, divide the square into three equal parts and label the parts "Left," "Centre," and "Right."

Step 3: Suppose that at the first branch in the maze, Reynaldo opens the left door and walks down the path. This path leads to two more doors. One door leads to Room A and the other leads to Room B. Reynaldo is equally likely to open either door. Therefore, divide the region labelled "Left" into two equal parts and label the parts "A" and "B."



Step 4: Suppose that at the first branch in the maze, Reynaldo opens the centre door and walks down the path. This path leads to Room B. Therefore, label the centre region "B."



Step 5: Suppose that at the first branch in the maze, Reynaldo opens the right door and walks down the path. This path leads to three more doors. One door leads to Room B and the other two doors lead to Room A.

A	B	
B		
B	A	A

Reynaldo is equally likely to open any of these doors. Therefore, divide the region labelled "Right" into three equal parts. Label one part "B" and the other two parts "A."

Step 6: The regions are of different sizes (see Step 5). Make the regions equal sizes by dividing the square into 18 equal parts. Relabel the diagram accordingly. **Hint:** The lowest common denominator of sixths, thirds, and ninths is eighteenths.

A	A	A	B	B	B
B	B	B	B	B	B
B	B	A	A	A	A

Step 7: Use the diagram in Step 6 to calculate the probability that Reynaldo will enter each room.

$$P(A) = \frac{\text{favourable}}{\text{possible}} \qquad P(B) = \frac{\text{favourable}}{\text{possible}}$$
$$= \frac{7}{18} \qquad = \frac{11}{18}$$

The probability Reynaldo will choose Room A is $\frac{7}{18}$.

The probability he will choose Room B is $\frac{11}{18}$.

Step 8: Use the probabilities to predict which room Reynaldo is more likely to enter.

$$\frac{11}{18} > \frac{7}{18}$$

Reynaldo is more likely to enter Room B.

14. Use objects such as sugar cubes to help you visualize the cubes. Put an ✗ on the faces that are painted.

Of the 27 cubes, 8 have three painted faces, 12 have two painted faces, 6 have one painted face, and 1 has no painted faces.

$$\therefore\ P(\textit{2 painted faces}) = \frac{12}{27}$$
$$= \frac{4}{9}$$

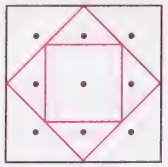
The probability of picking a cube with exactly two painted faces is $\frac{4}{9}$.

15. Use logical reasoning to determine the coins in the bag. Of the 6 coins in the bag, there are 3 nickels, 2 dimes, and 1 penny.

$$\therefore\ P(\textit{penny}) = \frac{1}{6}$$

The probability of choosing a penny is $\frac{1}{6}$.

16.



17. To solve this problem, you can use objects to represent the siblings.

a. S_1 S_2 S_3 B_1 B_2 J

✔ ✔ ✔

Jane's eldest sister (S_1) has 3 sisters.

b. S_1 S_2 S_3 B_1 B_2 J

✔ ✔ ✔ ✔

Jane's eldest brother (B_1) has 4 sisters.

18. Use logical reasoning to help you solve the problem.

Step 1: Make a chart showing how many brothers and sisters the **guest** could have. (The problem states that the **guest** has two more sisters than brothers.) **Hint:** You can use objects to help you make the chart.

Brothers	1	2	3	4	5
Sisters	3	4	5	6	7

Step 2: Assume the guest is a male. To test this assumption, make a chart showing how many sisters and brothers (including the guest) the **eldest sister** of the guest has. (The problem states that the guest's **eldest sister** has two more sisters than brothers.) **Hint:** Add 1 brother and subtract 1 sister from each column in the chart in Step 1.

Brothers	2	3	4	5	6
Sisters	2	3	4	5	6

The guest cannot be a male because each sister would then have an equal number of brothers and sisters.

The guest is female.

Step 3: Having concluded the guest is female, make a chart showing how many brothers and sisters (including the guest) the **eldest brother** could have. **Hint:** Subtract 1 brother and add 1 sister from each column in the chart in Step 1.

Brothers	0	1	2	3	4
Sisters	4	5	6	7	8

Step 4: Use the chart in Step 1 to calculate how many more sisters than brothers the **eldest brother** has.

$$4-0=4,\ 5-1=4,\ 6-2=4,\ldots$$

The eldest brother has 4 more sisters than brothers.

Section 1: Activity 2

1. a. Method 1: Using a Proportion

Step 1: Use this reasoning.

The ratio of the number of students favouring a candidate to the total number of students is the same in the sample and the total population. Write a proportion for each candidate.

Tiffany Chan

$$\frac{6}{30} = \frac{\quad}{300}$$

Marc Piard

$$\frac{7}{30} = \frac{\quad}{300}$$

Frank Wilson

$$\frac{4}{30} = \frac{\quad}{300}$$

Gillian Kowalchuk

$$\frac{13}{30} = \frac{\quad}{300}$$

Step 2: Find the missing term in each proportion.

Tiffany Chan

$$\frac{6}{30} \overset{\times 10}{=} \frac{60}{300}$$

About 60 students will vote for Tiffany.

Marc Piard

$$\frac{7}{30} \overset{\times 10}{=} \frac{70}{300}$$

About 70 students will vote for Marc.

Frank Wilson

$$\frac{4}{30} \overset{\times 10}{=} \frac{40}{300}$$

About 40 students will vote for Frank.

Gillian Kowalchuk

$$\frac{13}{30} \overset{\times 10}{=} \frac{130}{300}$$

About 130 students will vote for Gillian.

Method 2: Using a Fraction

Step 1: Use this reasoning.

Write the fraction of the sample who favoured each candidate.

In the sample, $\frac{6}{30}$ or $\frac{1}{5}$ favoured Tiffany Chan, $\frac{7}{30}$ favoured Marc Piard, $\frac{4}{30}$ or $\frac{2}{15}$ favoured Frank Wilson, and $\frac{13}{30}$ favoured Gillian Kowalchuk.

The same fraction of the total population is likely to favour these candidates.

In the total population, $\frac{1}{5}$ should favour Tiffany Chan, $\frac{7}{30}$ should favour Marc Piard, $\frac{2}{15}$ should favour Frank Wilson, and $\frac{13}{30}$ should favour Gillian Kowalchuk.

Step 2: Calculate the number of students who are likely to vote for each candidate.

Tiffany Chan

$$\frac{1}{\cancel{5}_{1}} \times \cancel{300}^{60} = 60$$

About 60 students should vote for Tiffany.

Marc Piard

$$\frac{7}{\cancel{30}_{1}} \times \cancel{300}^{10} = 70$$

About 70 students should vote for Marc.

Frank Wilson

$$\frac{2}{\cancel{15}_{1}} \times \cancel{300}^{20} = 40$$

About 40 students should vote for Frank.

Gillian Kowalchuk

$$\frac{13}{\cancel{30}_{1}} \times \cancel{300}^{10} = 130$$

About 130 students should vote for Gillian.

Note: You could also use a percent to calculate the number of votes for each student.

b. Based on the survey, Gillian Kowalchuk is likely to be elected president.

2. a. Based on the survey, the total population of junior high students is most likely to read the comics.

b. Based on the survey, the total population of junior high students is least likely to read the Business section.

c. 38% of 12 000

$= 0.38 \times 12\,000$

$= 4560$

About 4560 junior high students in the city are likely to read the Sports section.

d. 6% of 12 000

$= 0.06 \times 12\,000$

$= 720$

About 720 students are likely to read the Editorial/Letters to the Editor section.

3. a. 1% of 50 000

$= 0.01 \times 50\,000$

$= 500$

Of the people in this community, about 500 people are likely to have this blood type.

b. 37% of 50 000

$= 0.37 \times 50\,000$

$= 18\,500$

Of the people in this community, about 18 500 people are likely to have this blood type.

4. Step 1: Use this reasoning.

In the first sample, 200 trout were captured, tagged, and released.

In the second sample, 40 out of 250 trout, or $\frac{4}{25}$ of the sample, had been tagged the week before.

The ratio of the number of tagged trout to the total number of trout is the same in the second sample and the total population. Write a proportion.

$$\frac{\text{number of tagged trout}}{\text{total number of trout}} \qquad \frac{4}{25} = \frac{200}{\square}$$

Step 2: Find the missing term in the proportion.

$$\frac{4}{25} \overset{\times 50}{=} \frac{200}{1250}$$

There are about 1250 trout in the lake.

5. Step 1: Use this reasoning.

In the first sample, 150 golden eagles were captured, tagged, and released.

In the second sample, 125 of the 200 eagles, or $\frac{5}{8}$ of the eagles, were tagged the month before.

The ratio of the number of tagged eagles to the total number of eagles is the same in the second sample and the total population. Write a proportion.

$$\frac{\text{number of tagged eagles}}{\text{total number of eagles}} \qquad \frac{5}{8} = \frac{150}{\square}$$

Step 2: Find the missing term.

$$\frac{5}{8} \overset{\times 30}{=} \frac{150}{240}$$

There are about 240 golden eagles in the region.

6. Gerald won 24 out of 96 games. That is a relative frequency of about 0.25.

$$\therefore\ P(win) \doteq 0.25$$

The probability that Gerald will win the next video game he plays is about 0.25.

7. In the survey, 24 out of 30 people preferred hamburgers to hot dogs. That is a relative frequency of 0.8.

$\therefore P(\textit{hamburger}) \doteq 0.8$

The probability of a guest selecting a hamburger is about 0.8.

8. In the last year, Elaina has scored over 200 in 12 of 90 games. This is a relative frequency of about 0.13.

$\therefore P(\textit{scoring over 200}) \doteq 0.13$

The probability that Elaina will score over 200 in the next game she plays is about 0.13.

9. In the survey, 40 out of 50 families own a videocassette player. That is a relative frequency of 0.8.

$\therefore P(\textit{videocassette}) \doteq 0.8$

The probability that a family in the community owns a videocassette player is about 0.8.

10. $3\% = \dfrac{3}{100}$

The probability of Naomi winning the raffle is $\frac{3}{100}$.

11. $\dfrac{1}{4} = \dfrac{25}{100}$

$= 25\%$

The chances of Allanna winning the next chess game she plays is 25%.

12. a. $100\% = \dfrac{100}{100}$

$= 1$

The probability of rain on Friday is 1.

b. $80\% = \dfrac{80}{100}$

$= \dfrac{4}{5}$

The probability of rain on Saturday is $\frac{4}{5}$.

Now Try This

13. You can use the guess, check, and revise method to solve this problem. Answers will vary. This is one solution.

$\sqrt{9} \times 9 - \sqrt{9}$

$= 3 \times 9 - 3$

$= 27 - 3$

$= 24$

14. You may use a different point of view to solve this problem.

Step 1: Calculate the cost using the rate of 38 books at $12.50 per book.

$$38 \times 12.50 = 475$$

At this rate, 38 books cost $475.

Step 2: Calculate the cost using the rate of 41 books at $10.00 per book.

$$41 \times 10.00 = 410$$

At this rate, 41 books cost $410.

Step 3: Decide the minimum cost.

The minimum cost is $410.

Note: You save $65 and have three extra books!

Section 1: Activity 3

1. Methods may vary. Here is one solution.

Step 1: Analyse the problem and design a model that will help you solve this problem.

What is the probability that Jacques will win exactly 1 of his next 4 car races?

In the past, Jacques has won 1 out of every 6 car races in which he has competed.

You can use a standard die to simulate the outcome of each race. Let 1 (⚀) on the die represent a win. Let 2 (⚁), 3 (⚂), 4 (⚃), 5 (⚄), or 6 (⚅) represent a loss.

You can toss the die 4 times to simulate the 4 car races.

Step 2: Make a table like this.

	Outcomes			
	Toss 1	Toss 2	Toss 3	Toss 4
Trial 1				
Trial 2				
Trial 3				

Step 3: For the first trial, toss the die 4 times and record the outcomes in the table. Repeat for the second trial, third trial, and so on.

Answers will vary. Here is a table of possible outcomes.

	Outcomes			
	Toss 1	Toss 2	Toss 3	Toss 4
Trial 1	3	6	2	4
Trial 2	4	5	1	6
Trial 3	2	3	5	1

Step 4: Count the number of trials with favourable outcomes—trials in which **one** of the four tosses was a "1."

	Outcomes				Number of Favourable Outcomes
	Toss 1	Toss 2	Toss 3	Toss 4	
Trial 1	3	6	2	4	✗
Trial 2	4	5	1	6	✔
Trial 3	2	3	5	1	✔

Answers will vary. Here is one possible count.

In 100 trials there were 40 favourable outcomes.

Step 5: Calculate the relative frequency. Then use the relative frequency to estimate the probability that Jacques will win exactly 1 of his next 4 races.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{40}{100} = 0.4$$

Based on the simulation, the probability of Jacques winning exactly 1 of his next 4 races is about 0.4.

2. Methods may vary. Here is one solution.

Step 1: Analyse the problem and design a model that will help you solve this problem.

What is the probability that it will rain on Wednesday in 5 of the 8 widely separated cities?

There is a 50% chance of rain in each of the widely separated cities. This is a probability of $\frac{1}{2}$.

You can use a coin to simulate the chance of rain in each city. Let "Heads" represent rain and "Tails" represent no rain.

You can use 8 coins to simulate the 8 cities. (Alternatively, you can toss a single coin 8 times.)

Step 2: Make a table like this.

	Outcomes		
	Trial 1	Trial 2	Trial 3
Coin 1			
Coin 2			
Coin 3			
Coin 4			
Coin 5			
Coin 6			
Coin 7			
Coin 8			

Step 3: For the first trial, shake 8 coins in a container and spill the coins on a flat surface. Record the outcomes in the table. Repeat for the second trial, third trial, and so on.

Answers will vary. Here is a table of possible outcomes.

	Outcomes		
	Trial 1	Trial 2	Trial 3
Coin 1	Heads	Tails	Tails
Coin 2	Heads	Heads	Tails
Coin 3	Tails	Tails	Heads
Coin 4	Heads	Tails	Tails
Coin 5	Tails	Tails	Heads
Coin 6	Heads	Heads	Heads
Coin 7	Tails	Tails	Tails
Coin 8	Tails	Heads	Heads

Step 4: Count the number of trials with favourable outcomes—trials in which 5 of the 8 coins are "Heads."

Answers will vary. Here is one possible count.

In 100 trials, there were 20 favourable outcomes.

Step 5: Calculate the relative frequency. Then use the relative frequency to estimate the probability that it will rain in 5 of the 8 cities.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{20}{100} = 0.2$$

Based on the simulation, the probability that it will rain in 5 of the 8 cities is about 0.2.

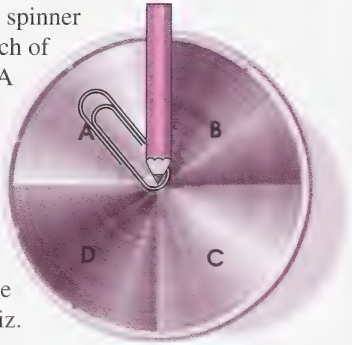
3. Methods may vary. Here is one solution.

Step 1: Analyse the problem and design a model that will help you solve this problem.

If Sandra guesses the answers, what is the probability that she will get 5 or more multiple-choice questions correct?

There are 10 multiple-choice questions on the quiz. For each question, there are four possible outcomes—1 correct answer and 3 wrong answers.

You can use a dial and spinner like this to simulate each of Sandra's guesses. Let A represent a correct answer. Let B, C, and D represent a wrong answer.



Twirl the spinner 10 times to simulate the 10 questions on the quiz.

Step 2: Make a table like this.

	Outcomes		
	Trial 1	Trial 2	Trial 3
Spin 1			
Spin 2			
Spin 3			
Spin 4			
Spin 5			
Spin 6			
Spin 7			
Spin 8			
Spin 9			
Spin 10			

Step 3: For the first trial, twirl the spinner 10 times and record the outcomes in the table. Repeat for the second trial, third trial, and so on.

Step 4: Count the number of trials with favourable outcomes—trials with 5 or more *A*s.

Answers will vary. Here is one possible count.

In 100 trials, there were 15 favourable outcomes.

Step 5: Calculate the relative frequency. Then use the relative frequency to estimate the probability that Sandra will get 5 or more correct answers.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{15}{100} = 0.15$$

Based on the simulation, the probability that Sandra will get 5 or more correct answers is about 0.15.

4. Methods may vary. Here is one solution.

Step 1: Analyse the problem and design a model that will help you solve this problem.

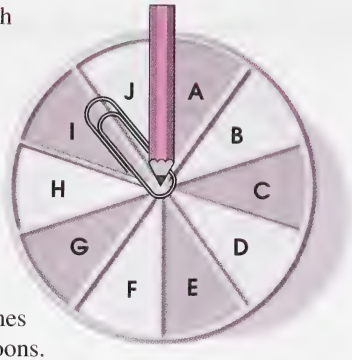
What is the probability of blowing up 6 balloons in a row without bursting any?

There is a 10% chance that a balloon will burst before it is fully inflated.

$$10\% = \frac{10}{100}$$
$$= \frac{1}{10}$$

Therefore, the probability that a balloon will burst before it is fully inflated is $\frac{1}{10}$.

You can use a dial with 10 sectors like this to simulate a balloon being inflated. Let A represent a burst balloon. Let B, C, D, E, F, G, H, I, and J represent a balloon that does not burst.



Twirl the spinner 6 times to represent the 6 balloons.

Step 2: Make a table like this.

	Outcomes		
	Trial 1	Trial 2	Trial 3
Spin 1			
Spin 2			
Spin 3			
Spin 4			
Spin 5			
Spin 6			

Step 3: For the first trial, twirl the spinner 6 times and record the outcomes in the table. Repeat for the other trials.

Step 4: Count the number of trials with favourable outcomes—trials in which there are no *A*s.

Answers will vary. Here is one possible count.

In 100 trials, there were 55 trials with favourable outcomes.

Step 5: Calculate the relative frequency. Use the relative frequency to estimate the probability of inflating 6 balloons in a row without bursting any.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{55}{100} = 0.55$$

Based on the simulation, the probability of blowing up 6 balloons in a row without bursting any is about 0.55.

5. Methods may vary. Here is one possible solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet to help you solve the problem.

What is the probability that a two-stage rocket will have a successful flight?

In the past, each stage fired 3 out of 4 times.

Use the formula $= \text{RAND}(4)$ to simulate the result of a stage firing. Let 1, 2, and 3 represent a success. Let 4 represent a failure. Use two cells to represent the 2 stages.

Step 2: Open a new spreadsheet file and use row 1 to show the results of the first trial. Enter = RAND(4) in cells A1 and B1.

Random numbers (1, 2, 3, or 4) will appear in the cells.

Step 3: To show the results of other trials in rows 2, 3, 4, 5, and so on, highlight cell A1 and shift-click on cell B100.

Then click and hold on "Calculate" in the menu bar. A pop-up menu will appear.

Continue holding and drag down to "Fill Down."

Random numbers will appear in columns A and B of the first 100 rows. Print the page.

Step 4: Count the number of rows without a 4.

Answers will vary. Here is one possible count.

In 100 rows, there were 60 rows without a 4.

Step 5: Calculate the relative frequency. Use the relative frequency to estimate the probability that the rocket will have a successful flight.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{60}{100} = 0.6$$

Based on the simulation, the probability that the rocket will have a successful flight is about 0.6.

6. Methods may vary. Here is one possible solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet to help you solve the problem.

What is the probability that in a bottle of 20 antacid tablets, there will be two or more ineffective dosages?

Based on the sample, the antacid tablets fall below prescribed strength 3 out of 50 times.

Use the formula = RAND(50) to simulate the strength of the tablets. Let 1, 2, and 3 represent tablets that are below prescribed strength. Let 4, 5, …, 50 represent tablets that are at the prescribed strength.

Use 20 cells to represent the 20 tablets.

Step 2: Open a new spreadsheet file and use row 1 to show the results of the first trial.

Enter = RAND(50) in cells A1 to S1. **Hint:** Enter = RAND(50) in cell A1. Then highlight cell A1 and shift-click on cell S1. Then click and hold on "Calculate" in the menu bar. Continue holding and drag down to "Fill Right."

Random numbers will appear in the first 20 cells of row 1.

Step 3: To simulate 100 trials, highlight cell A1 and shift-click on cell S100. Then click and hold on "Calculate" in the menu bar. Continue holding and drag down to "Fill Down."

Random numbers will appear in the first 20 cells of 100 rows. (**Note:** You may wish to narrow the column widths to make reading easier.) Print the page(s).

Step 4: Count the number of rows with two or more of 1, 2, or 3.

Answers will vary. Here is one possible count.

In 100 rows, there were 26 rows with two or more of 1, 2, or 3.

Step 5: Calculate the relative frequency. Use the relative frequency to estimate the probability that a bottle of 20 antacid tablets will have two or more ineffective dosages.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{26}{100} = 0.26$$

Based on the simulation, the probability that in a bottle of 20 antacid tablets there will be two or more ineffective dosages is about 0.26.

7. Methods may vary. Here is one solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet to help you solve the problem.

What is the probability that if two accidents occur in the city during a certain week, they will both occur on the weekend?

Records show that in this city car accidents are equally likely to happen on each day of the week.

Use the formula $= \text{RAND}(7)$ to simulate the 7 days of the week. Let 1 be Monday, 2 be Tuesday, 3 be Wednesday, 4 be Thursday, 5 be Friday, 6 be Saturday, and 7 be Sunday.

Use two cells to represent the 2 accidents.

Step 2: Open a new spreadsheet file and use row 1 to show the results of the first trial.

Enter the formula $= \text{RAND}(7)$ in cells A1 and B1.

Random numbers (either 1, 2, 3, 4, 5, 6, or 7) will appear.

Step 3: To simulate 100 trials, highlight cell A1 and shift-click on cell B100. Then click and hold on "Calculate" in the menu bar. Continue holding and drag down to "Fill Down."

Random numbers will appear in the first 2 cells of 100 rows. Print the page(s).

Step 4: Count the number of rows with 6 and/or 7.

Answers will vary. Here is one possible count.

In 100 rows, there were 8 rows with 6 and/or 7.

Step 5: Calculate the relative frequency. Use the relative frequency to estimate the probability that both accidents will be on the weekend.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{8}{100} = 0.08$$

Based on the simulation, the probability that if two accidents occur in the city during a certain week, they will both occur on the weekend is about 0.08.

8. Methods may vary. Here is one possible solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet to help you solve the problem.

If a researcher forms six pairs by picking people at random, about how many pairs would you expect to consist of one introvert and one extrovert?

In the general population, the ratio of introverts to extroverts to ambiverts is 1: 1: 1.

$1 + 1 + 1 = 3$

Therefore, 1 in 3 is an introvert, 1 in 3 is an extrovert, and 1 in 3 is an ambivert.

Use the formula = RAND(3) to simulate the personality types. Let 1 be an extrovert. Let 2 be an introvert. Let 3 be an ambivert.

Use 12 cells to represent the 6 pairs.

Step 2: Open a new spreadsheet file and use row 1 to show the results of the first trial.

Enter the formula = RAND(3) in cells A1 and B1, D1 and E1, G1 and H1, J1 and K1, M1 and N1, and P1 and Q1. **Hint:** Leave blank cells between each pair of cells to make counting easier later.

Random numbers (either 1, 2, or 3) will appear in these cells.

Step 3: To simulate 100 trials, highlight cell A1 and shift-click on cell Q100. Then click and hold on "Calculate" in the menu bar. Continue holding and drag down to "Fill Down."

Random numbers will appear in the 6 pairs of cells in the 100 rows. Print the page(s).

Step 4: Count the number of times a 1 (for extrovert) and 2 (for introvert) appear in a pair of cells in each row.

Answers will vary. Here is one possible count.

In 100 rows, a 1 and 2 occurred 300 times.

Step 5: Find the average number of times that a 1 and 2 occurred per trial. Use this average to estimate how many pairs you would expect to consist of one introvert and one extrovert.

Answers will vary. This solution is based on the count in Step 4.

$$\frac{133}{100} = 1.33$$

Based on the simulation, you would expect about 1.33 pairs out of the 6 pairs to consist of one introvert and one extrovert.

9. Methods may vary. Here is one possible solution using *ClarisWorks*™.

Step 1: Analyse the problem and use a spreadsheet program to help you solve the problem.

About how many of the next five applicants will pass both parts of the entrance test?

On the academic part of the test, 5 out of 6 applicants usually pass. Use the formula $= \text{RAND}(6)$ to represent the passes and failures. Let 1, 2, 3, 4, and 5 represent a pass. Let 6 represent a failure.

On the athletic ability part of the test, 1 in every 2 applicants usually passes. Use the formula $= \text{RAND}(2)$ to represent the passes and failures. Let 1 represent a pass. Let 2 represent a failure.

Use five pairs of cells to represent the two parts of the test that the 5 applicants will take.

Step 2: Open a new spreadsheet file and use row 1 to show the results of the first trial.

Enter the formula $= \text{RAND}(6)$ in cells A1, D1, G1, J1, and M1. Random numbers (1, 2, 3, 4, 5, or 6) will appear in these cells.

Enter the formula $= \text{RAND}(2)$ in cells B1, E1, H1, K1, and N1. A random number (either 1 or 2) will appear in these cells.

(There will be an empty column between each pair of numbers.)

Step 3: To simulate 100 trials, highlight cell A1 and shift-click on cell N100. Then click and hold on "Calculate" in the menu bar. Continue holding and drag down to "Fill Down."

Random numbers will appear in the 100 rows. Print the page(s).

Step 4: In each row, count the number of times a 1, 2, 3, 4, or 5 appears in the first cell of each pair of cells, together with a 1 in the second cell of the pair. (You may find it helpful to cross out or highlight the inappropriate pairs.)

Answers will vary. Here is one possible count.

In 100 rows, there was a total of 210 times when a 1, 2, 3, 4, or 5 appeared in the first cell of a pair of cells, together with a 1 in the second cell of the pair.

Step 5: Find the average. Use the average to estimate the number of applicants who will pass both parts of the test.

Answers will vary. This solution is based on the count in Step 4.

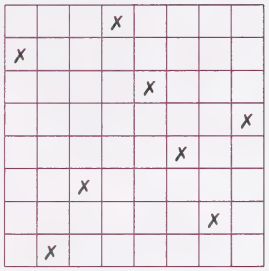
$$\frac{210}{100} = 2.1$$

Based on the simulation, you would expect about 2.1 applicants out of the five applicants to pass both parts of the test.

Now Try This

10. You may use a real checkerboard and checkers to help answer this question.

This is one possible answer. The following diagram represents the checkerboard. The position of each checker is represented by an ✗.



Section 1: Follow-up Activities

Extra Help

1. Yes, Sharon's reasoning is flawed. She is using reasoning called the "gambler's fallacy." Sharon reasons that a girl is more likely to be born so the sample will more closely reflect the theoretical 50% girls and 50% boys. Actually, with each birth there is an equal chance of a boy or a girl.

2. Answers will vary.

One reason that the polls and election results could have varied so greatly is that voter turnout may have been low.

Another reason is that people could have voted strategically. That is, they could have changed their vote so that the candidates leading in the polls were defeated.

3. Julia's prediction is more likely to be a better indicator of the probability of a rainy day.

The law of large numbers states that the greater the number of trials you conduct in an experiment, the closer the experimental result will be to the theoretical results. Pat based her prediction on only 30 days, which is not a very large set of data. Julia, on the other hand, based her prediction on 50 years of data for the month of April.

4. When the boy tire maker married the girl tire maker, what did everyone say? THEY MAKE A NICE SPARE. **Note:** There are no letters for the $\frac{1}{8}$, $\frac{2}{13}$, $\frac{1}{5}$, and 32 spaces.

Enrichment

1. **Step 1:** Use this reasoning.

In Canadian federal elections, 3 out of 4 eligible voters usually cast their votes. This means that 3 cast their votes for every 1 that does not.

Step 2: Calculate the odds in favour of a person voting.

$$\text{Odds in favour} = \frac{\text{favourable outcomes}}{\text{unfavourable outcomes}}$$
$$= \frac{3}{1}$$

The odds in favour of a person voting are 3 to 1.

2. $$\text{Odds in favour} = \frac{\text{favourable outcomes}}{\text{unfavourable outcomes}}$$
$$= \frac{17}{13}$$

The odds in favour of the chosen person being a woman are 17 to 13.

3. **Step 1:** Use this reasoning.

A mother gives birth to twins in 1 out of 96 births. This means that for every 95 births that do not result in twins there will be 1 birth that results in twins.

$96 - 1 = 95$

Step 2: Calculate the odds against a mother having twins.

$$\text{Odds against twins} = \frac{\text{unfavourable outcomes}}{\text{favourable outcomes}}$$
$$= \frac{95}{1}$$

The odds against a mother having twins are 95 to 1.

Did You Know?

4. **a.** Canadian Philip McGinnis invented the web-barrier.

b. The web-barrier provided a more efficient and fairer way to start horse races than the previously used methods.

Section 2: Activity 1

1. Answers will vary; possible answers follow.

a. You could keep a record of how many showers or baths were required to consume a bar of soap. This would be an experiment.

b. You could stand at an intersection and count the number of cars turning left. This would be a survey.

c. You could interview all the dentists or a sample of the dentists. This would be a survey.

d. You could collect rainfall daily and record the amount. This would be an experiment.

e. You could interview the members of your family. This would be a survey.

f. You could conduct a taste-testing experiment or a survey.

2. Answers will vary; possible answers follow.

a. You could use an almanac such as *The Information Please Sports Almanac*, a newspaper, or the Internet.

b. You could use a book such as *The Guinness Book of Records*, an almanac such as *The World Almanac® and Book of Facts*, or an encyclopedia.

c. You could use an encyclopedia, an almanac such as *The Canadian Global Almanac*, or the Internet.

d. You could use a consumer magazine or the Internet.

e. You could use a newspaper or the Internet.

3. The advantage of using primary data is that you know how the data were collected. One disadvantage is that your experience in data collection is limited. Collecting data directly is also time-consuming and expensive.

One advantage of using secondary data is that you rely on professionals who have experience in data collection. Collecting data indirectly is also cheaper and quicker. Another disadvantage is that you are not always aware of how the data were collected.

4. One advantage of a personal interview is that this kind of survey is taken seriously by respondents and there is usually a good response rate. One disadvantage is that if people are not at home, the interviewer may need to make a second visit. Interviewing the respondents in person is very time-consuming and expensive.

One advantage of a telephone survey is that it is usually easier to contact people. If people are not at home, the call can be repeated easily at a later time. However, people decline to participate in a telephone survey more than in a personal interview.

One advantage of mailing a questionnaire is that it is less costly because people do not have to be trained to administer the questionnaire. However, mail surveys are often viewed as "junk mail" and are discarded, so fewer people participate in the poll.

5. Answers will vary. Here are three possible ways.

- You could assign numbers to the 100 people. Then you could use a table of random numbers generated by a computer to select the sample of 10 people.

- You could assign numbers to the 100 people. Then you could number 100 ping-pong balls, mix the balls thoroughly, and use the balls to select the sample of 10 people.

- You could arrange the list of 100 names in order alphabetically and then choose every 10th person in the list.

6. a. Answers will vary. Bias can be avoided by choosing 10 items produced at different times and by different workers.

b. Answers will vary. A telephone directory can be used to select a random sample. By selecting 10 pages at random and taking the first telephone numbers that end in 3 (or some other number chosen at random) from each page, you should obtain an unbiased sample.

7. a. The target group was passengers of long airplane flights.

b. Yes, the services would probably be ranked differently for short airplane flights. People who have never flown may not be aware of what services would be most appreciated.

8. a. Scientists might be biased in favour of using animals for laboratory testing. Animal-rights groups may be biased against using the animals.

b. Civil-rights lawyers may be biased against censorship. Parents may be biased in favour of censorship to protect their children.

c. Right-to-life groups may be biased in favour of the rights of the unborn. Right-to-choice groups may be biased in favour of the mother's right to choose.

9. You must be careful to write clear and readable questions so that there are no misunderstandings.

You must be sensitive to personal and cultural beliefs so that respondents will not be offended.

You must be careful not to make the questionnaire too long.

10. a. These two questions could be used because they are not "loaded." The wording does not attempt to influence the respondents.

- Do you think the number of VLTs in Alberta is too low, about right, or too high?
- Are you in favour of banning VLTs in Alberta?

The other two questions are biased.

b. The question was a "good question" for the following reasons.

- It was not "loaded."
- The wording does not attempt to influence the respondents.
- The question is clearly worded.
- A factual (yes or no) answer can be given.

Now Try This

11. Use truth tables to answer the problem.

Step 1: Assume that A is the true statement. Put T for true under A in the table.

A	B	C	D	E	F
T					

- B contradicts A; therefore, put F under B.

A	B	C	D	E	F
T	F				

- C confirms A is true; therefore, put T under C.

A	B	C	D	E	F
T	F	T			

One and only one statement is true; therefore, A can be eliminated as the true statement.

Step 2: Assume that B is the true statement.

- Put F under A (you eliminated it as the true statement in Step 1). Put T for true under B in the table.

A	B	C	D	E	F
F	T				

- C confirms B is true; therefore, put T under C.

A	B	C	D	E	F
F	T	T			

One and only one statement is true; therefore, B can be eliminated as a true statement.

Step 3: Use this reasoning. You have eliminated A and B; therefore, C cannot be true.

Eliminate C.

Step 4: Use this reasoning. You have eliminated A, B, and C as the true statements; therefore, D cannot be true.

Eliminate D.

Step 5: Use this reasoning. You have concluded that A, B, C, and D are false; therefore, E is true. Put F under A, B, C, and D. Put T under E.

A	B	C	D	E	F
F	F	F	F	T	

If E is true, then F must be false. Put F under F.

A	B	C	D	E	F
F	F	F	F	T	F

The true statement is E.

Section 2: Activity 2

1. Arrange the data in ascending order.

10.8	10.8	10.9	10.9	11.0
11.0	11.0	11.1	11.1	

a. The lower extreme of Susan Humphreys's scores is 10.8. The upper extreme is 11.1.

b. $11.1 - 10.8 = 0.3$

The range of Susan Humphreys's scores is 0.3.

2. a. The lower extreme is 0 satellites per planet.

b. The upper extreme is 18 satellites per planet.

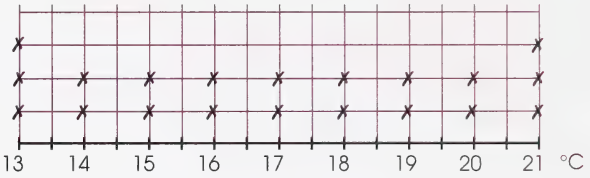
c. $18 - 0 = 18$

The range is 18 satellites per planet.

d. Yes, there are two clusters. One cluster is from 0 to 2 inclusive. The other cluster is from 15 to 18 inclusive.

e. Yes, there are two gaps in the data. One gap is from 3 to 7 inclusive. The other is from 9 to 14 inclusive.

3. a.



b. The lower extreme is 13°C.

c. The upper extreme is 21°C.

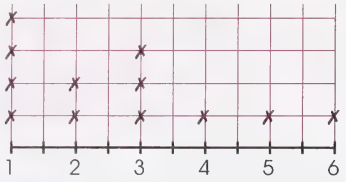
d. $21 - 13 = 8$

The range is 8°C.

e. No, the values are evenly distributed.

f. No, the values are evenly distributed.

4. a. Following is the line plot of the numbers Frank rolled.



b. The lower extreme is 1.

c. The upper extreme is 6.

d. $6 - 1 = 5$

The range is 5.

e. Yes, there is a cluster between 1 and 3 inclusive.

f. No, there are no gaps in the data.

5. a. The lower extreme is 39 years.

b. The upper extreme is 74 years.

c. $74 - 39 = 35$

The range is 35 years.

d. Yes, there is a cluster between 45 years and 59 years inclusive.

e. Yes, there is a gap from 40 years to 44 years.

6. a. **Step 1:** Record the data in a stem and leaf plot.

Stem	Leaf
5	0
•	9
6	3, 3
•	9, 6
7	1, 0, 4, 4
•	6, 5, 5, 9, 8, 8
8	2, 2
•	6
9	0

Step 2: Arrange the leaves in ascending order.

Stem	Leaf
5	0
•	9
6	3, 3
•	6, 9
7	0, 1, 4, 4
•	5, 5, 6, 8, 8, 9
8	2, 2
•	6
9	0

b. The lower extreme is 50 h.

c. The upper extreme is 90 h.

d. $90 - 50 = 40$

The range is 40 h.

e. Yes, there is a cluster in the 70s.

f. No, there are no gaps in the data.

7. a. **Step 1:** Arrange the data in ascending order.

19	20	21	23	23	23	23
24	25	25	26	26	28	

Step 2: Find the median.

median

19	20	21	23	23	23	(23)
24	25	25	26	26	28	

The median is 23 g per serving.

Step 3: Find the quartiles.

$$\frac{21+23}{2} = \frac{44}{2} = 22$$

lower quartile

19	20	21	23	23	23	(23)
24	25	25	26	26	28	

upper quartile

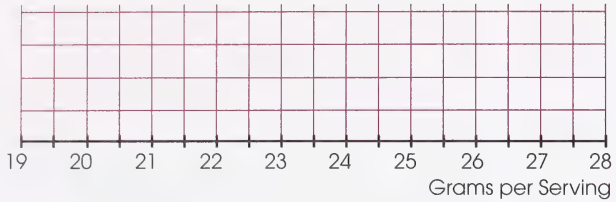
$$\frac{25+26}{2} = \frac{51}{2} = 25.5$$

The lower quartile is 22 g per serving; the upper quartile is 25.5 g per serving.

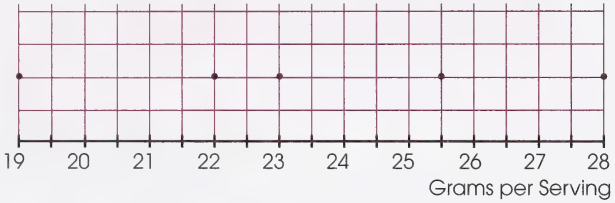
Step 4: Find the extremes.

The lower extreme is 19 g per serving; the upper extreme is 28 g per serving.

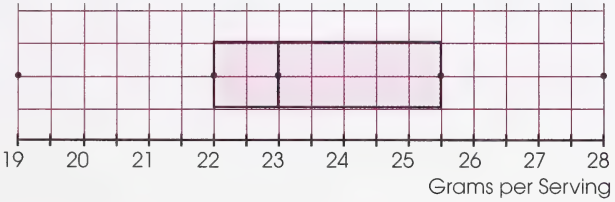
Step 5: Draw a number line that will include the extremes.
Note: Choose the appropriate graph paper and scale for the number line.



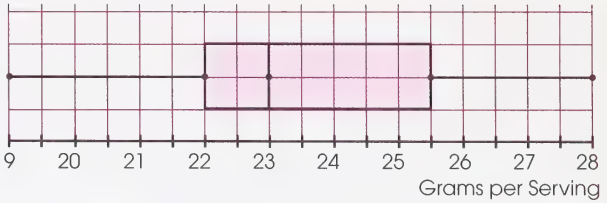
Step 6: Plot the median, quartiles, and extremes above the number line.



Step 7: Draw a box using the lower and upper quartiles as the ends of the box. Draw a vertical line through the box at the median.



Step 8: Draw whiskers (horizontal lines) from the box to the extremes.



b. $28 - 19 = 9$

The range is 9 g per serving.

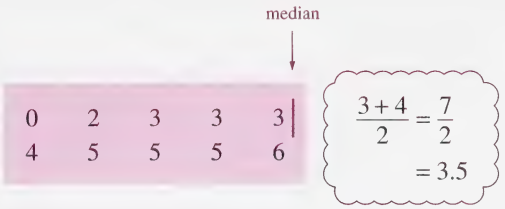
c. $25.5 - 22 = 3.5$

The interquartile range is 3.5 g per serving.

8. a. Step 1: Arrange the data in ascending order.

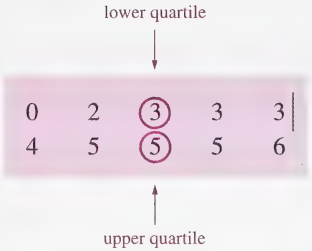
0	2	3	3	3
4	5	5	5	6

Step 2: Find the median.



The median is 3.5 h per day.

Step 3: Find the quartiles.

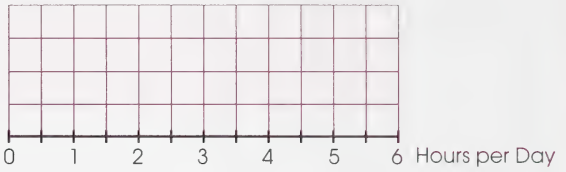


The lower quartile is 3 h per day; the upper quartile is 5 h per day.

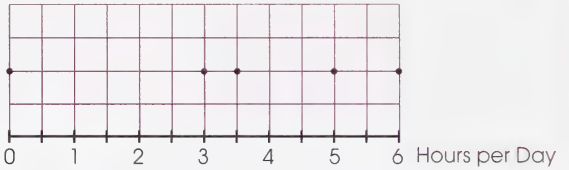
Step 4: Find the extremes.

The lower extreme is 0 h per day; the upper extreme is 6 h per day.

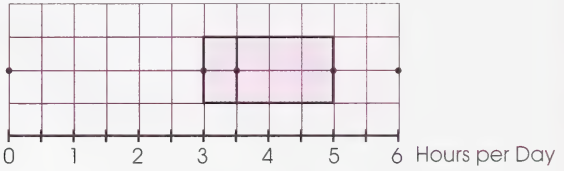
Step 5: Draw a number line that will include the extremes.
Note: Choose the appropriate graph paper and scale for the number line.



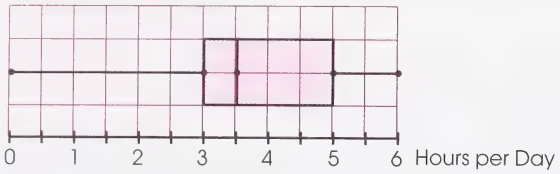
Step 6: Plot the median, lower and upper quartiles, and lower and upper extremes above the number line.



Step 7: Draw a box using the lower and upper quartiles as the ends of the box. Draw a vertical line through the box at the median.



Step 8: Draw the whiskers from the box to the extremes.



b. $6 - 0 = 6$

The range is 6 h per day.

c. $5 - 3 = 2$

The interquartile range is 2 h per day.

9. a. Step 1: Arrange the data in ascending order.

100 105 110 115 120 135 140

Step 2: Find the median.

100 105 110 115 120 135 140

median

The median is \$115.

Step 3: Find the lower and upper quartiles.

100 105 110 115 120 135 140

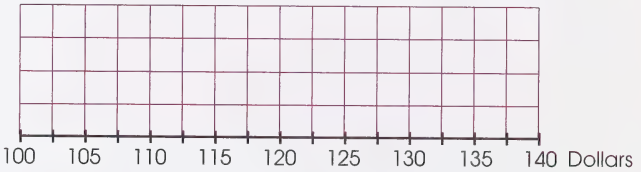
lower quartile median upper quartile

The lower quartile is \$105; the upper quartile is \$135.

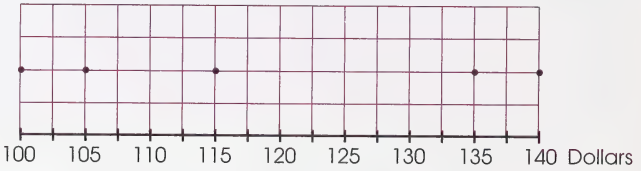
Step 4: Find the extremes.

The lower extreme is \$100; the upper extreme is \$140.

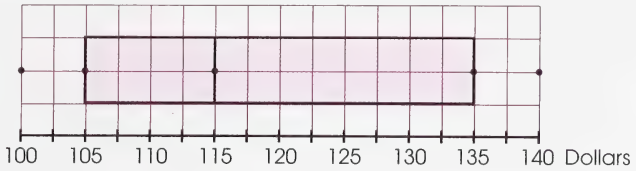
Step 5: Draw a number line that will include the extremes.
Note: Choose the appropriate graph paper and scale for the number line.



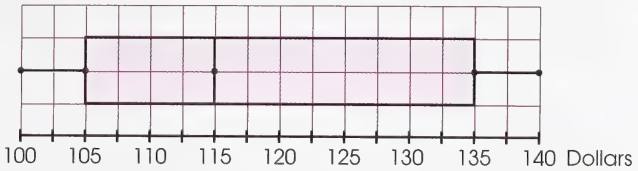
Step 6: Plot the median, lower and upper quartiles, and lower and upper extremes above the number line.



Step 7: Draw a box using the quartiles as ends of the box. Draw a vertical line through the box at the median.



Step 8: Draw the whiskers from the box to the extremes.



b. $140 - 100 = 40$

The range is \$40.

c. $135 - 105 = 30$

The interquartile range is \$30.

10. a. A stem and leaf plot shows all the values. You can see gaps and clusters in the values.

b. A box and whisker plot shows only certain statistics—the extremes, quartiles, and median. You can easily calculate the interquartile range from a box and whisker plot.

11. a. Step 1: Find the median score.

Stem	Leaf
6	7, 7, 9
7	1, 2, 4, 8, 9, 9
8	4, 5, 7, (8), 9
9	2, 3, 4, 4, 6, 7, 8
10	0, 2, 4, 6

The median score is 88 strokes.

Step 2: Find the lower and upper quartiles.

Stem	Leaf
6	7, 7, 9
7	1, 2, 4, |8, 9, 9
8	4, 5, 7, (8), 9
9	2, 3, 4, 4, 6, |7, 8
10	0, 2, 4, 6

$\frac{74 + 78}{2} = 76$

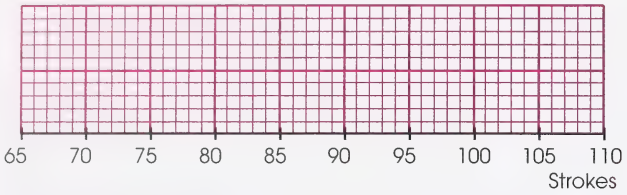
$\frac{96 + 97}{2} = 96.5$

The lower quartile is 76 strokes; the upper quartile is 96.5 strokes.

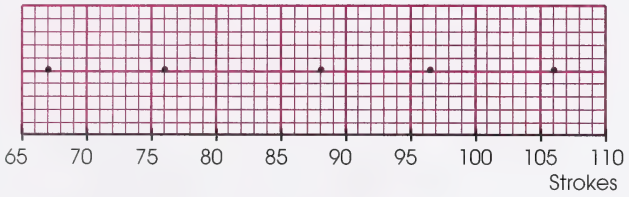
Step 3: Find the lower and upper extremes.

The lower extreme is 67 strokes; the upper extreme is 106 strokes.

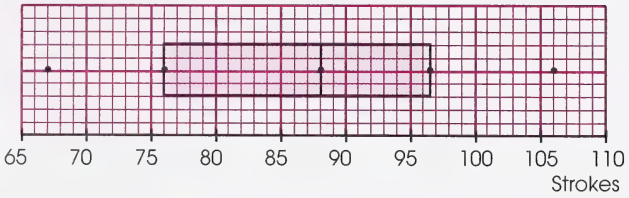
Step 4: Draw a number line that includes the extremes.
Note: Choose the appropriate graph paper and scale.



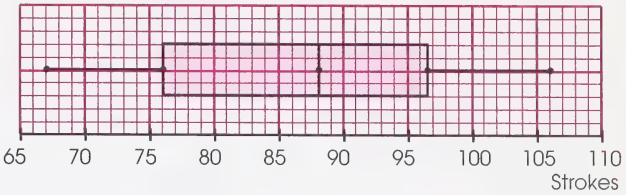
Step 5: Plot the median, lower and upper quartiles, and lower and upper extremes above the number line.



Step 6: Draw a box using the quartiles as ends of the box. Draw a vertical line through the box at the median.



Step 7: Draw the whiskers from the box to the extremes.



b. $106 - 67 = 39$

The range of the scores is 39 strokes.

c. $96.5 - 76 = 20.5$

The interquartile range of the scores is 20.5 strokes.

12. a. Step 1: Find the median.

Stem	Leaf
11	0, 1, 8
12	4, 5, 9, 9
13	0, 0, 1, 4, 5, 7, 7, 7, 9, 9
14	(0), 2, 2, 2, 3, 5, 5, 5, 5, 6
15	0, 2, 2, 7, 8, 9, 9, 9

The median number of trash cans filled per business is 140.

Step 2: Find the lower and upper quartiles.

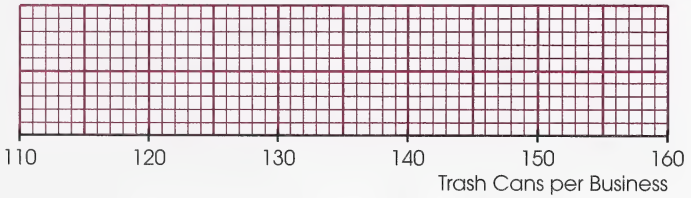
Stem	Leaf
11	0, 1, 8
12	4, 5, 9, 9
13	0, (0), 1, 4, 5, 7, 7, 7, 9, 9
14	(0), 2, 2, 2, 3, 5, 5, 5, 5, (6)
15	0, 2, 2, 7, 8, 9, 9, 9

The lower quartile is 130; the upper quartile is 146.

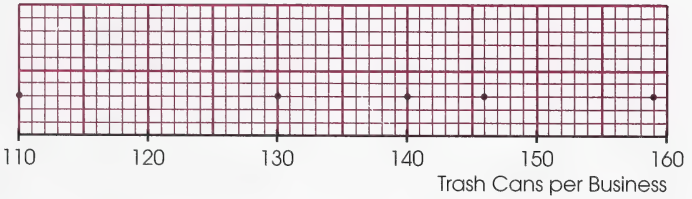
Step 3: Find the lower and upper extremes.

The lower extreme is 110; the upper extreme is 159.

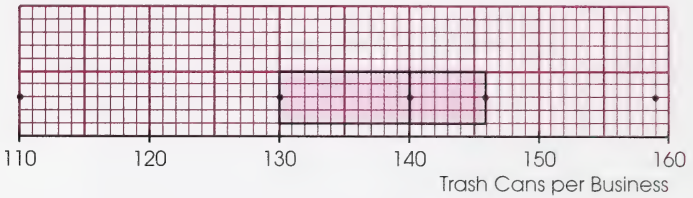
Step 4: Make a number line that includes the extremes.



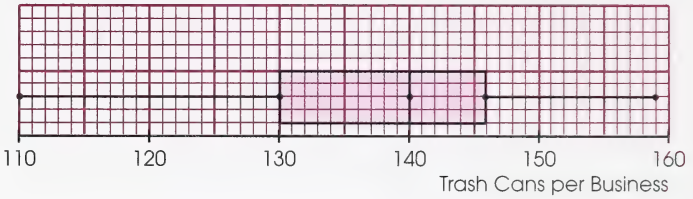
Step 5: Plot the median, quartiles, and extremes.



Step 6: Draw a box using the quartiles as the ends of the box. Draw a vertical line through the box at the median.



Step 7: Draw the whiskers from the box to the extremes.



b. $159 - 110 = 49$

The range is 49 trash cans per business.

c. $146 - 130 = 16$

The interquartile range is 16 trash cans per business.

Did You Know?

13. a. A Canadian, Frank Pomp, invented garbage bags.

b. Garbage bags are more convenient than the cans. They are less messy and easier to handle. Since bags are tied, odours are sealed in and they don't attract insects, cats, and dogs as much as garbage cans did.

c. Green was chosen to blend in with the lawns.

d. Answers will vary. Blue garbage bags are often used to indicate recyclable materials. Clear bags are most often used for plant material—grass clippings, leaves, twigs, etc.

Now Try This

14. a. **Step 1:** Find the amount generated by the bottles and cans.

$$675 \times 0.15 = 101.25$$

The bottle and can returns generated $101.25.

Step 2: Find the amount generated by selling the paper.

$$568.5 \times 0.052 = 29.562$$

The paper generated $29.56.

Step 3: Find the amount generated by selling the scrap metal.

$$138.5 \times 4.25 = 588.625$$

Selling the scrap metal generated $588.63.

Step 4: Find the total amount.

$$101.25 + 29.56 + 588.63 = 719.44$$

The total made by the group was $719.44.

b. $719.44 \div 5 = 143.888$

$\doteq 143.89$

On average, each person's contribution was about $143.89.

15. a. $4^2 = 16$

The last digit is 6.

b. $4^3 = 64$

The last digit is 4.

c. $4^4 = 256$

The last digit is 6.

d. $4^5 = 1824$

The last digit is 4.

e. Use patterns to solve the problem. The standard form of powers of 4 with an odd exponent end in 4. The standard form of powers of 4 with an even exponent end in 6.

The last digit of the standard form of 4^{40} is 6.

Section 2: Activity 3

1. The mode of the sample is 19 haircuts. Long straight hair occurred most frequently in the sample. You can think of the mode of this set of data as the haircut that was most fashionable among twelfth-grade females in 1970.

2. a. $\frac{3+5}{2} = \frac{8}{2} = 4$

The median age is 4 years.

b. $\frac{1+2+3+5+10+20}{6} = \frac{41}{6}$

$\doteq 6.8$

The mean is about 6.8 years.

c. There is no mode.

3. a.

0	0	2	(2)	2	3	5

median

The median is 2 accidents per day.

b. $\frac{0+0+2+2+2+3+5}{7} = \frac{14}{7}$

$= 2$

The mean is 2 accidents per day.

c. The mode is 2 accidents per day.

4. a.

1	1	1	1	1	2	2	(3)
4	4	5	6	9	10	10	

median

The median response time is 3 min.

b. $\frac{5\times1+2\times2+3+2\times4+5+6+9+2\times10}{15}$

$= \frac{5+4+3+8+5+6+9+20}{15}$

$= \frac{60}{15}$

$= 4$

The mean response time is 4 min.

c. The mode response time is 1 min.

5. After the bonus points are added, the set of values becomes this.

3 4 4 6 9 10

a. The mode before the bonus points are added is 1.

The mode after the bonus points are added is 4.

b. The median before the bonus points are added is 2.

$$\frac{1+3}{2} = 2$$

0 1 1 | 3 6 7

The median after the bonus points are added is 5.

$$\frac{4+6}{2} = 5$$

3 4 4 | 6 9 10

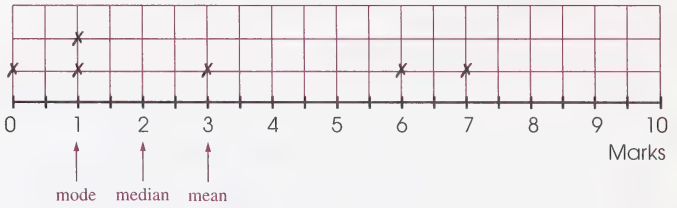
c. The mean before the bonus points are added is 3.

$$\frac{0+1+1+3+6+7}{6} = 3$$

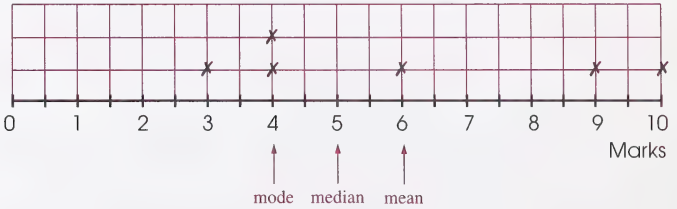
The mean after the bonus points are added is 6.

$$\frac{3+4+4+6+9+10}{6} = 6$$

d. The line plot of the scores before the bonus points are added follows.



e. The line plot of the scores after the bonus points are added follows.



f. The addition of the bonus points causes the scores to slide to the right on the line plot. The mode, median, and mean are each 3 points higher after the bonus points are added.

6. The new set of values becomes this.

13.10 13.35 14.15 14.15 14.60 15.35

a. The mode price before the price decrease is $15.80.

The mode price after the price decrease is $14.15.

b. The median price before the price decrease is \$15.80.

The median price after the price decrease is \$14.15.

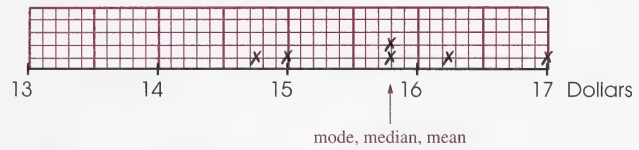
c. The mean price before the price decrease is about \$15.80.

$$\frac{14.75+15.00+2\times15.80+16.25+17.00}{6}\doteq 15.80$$

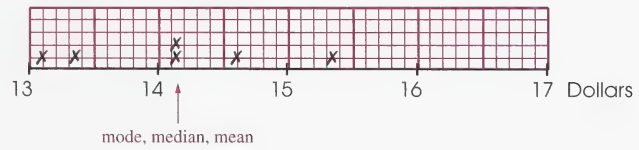
The mean price after the price decrease is about \$14.15.

$$\frac{13.10+13.35+2\times14.15+14.60+15.35}{6}\doteq 14.15$$

d. Following is the line plot before the price decrease.



e. Following is the line plot after the price decrease.



f. The price decrease causes the data to slide to the left. The mode, median, and mean are each \$1.65 less after the decrease.

7. The set of prices for 1997 follows.

1.20	1.40	1.52	1.60	1.60
1.68	1.80	1.80	1.80	2.00

a. The mode of the prices in 1970 is \$0.45.

The mode of the prices in 1997 is \$1.80.

b. The median of the prices in 1970 is \$0.41.

0.30	0.35	0.38	0.40	0.40
0.42	0.45	0.45	0.45	0.50

$$\frac{0.40+0.42}{2}=0.41$$

The median of the prices in 1997 is \$1.64.

1.20	1.40	1.52	1.60	1.60
1.68	1.80	1.80	1.80	2.00

$$\frac{1.60+1.68}{2}=1.64$$

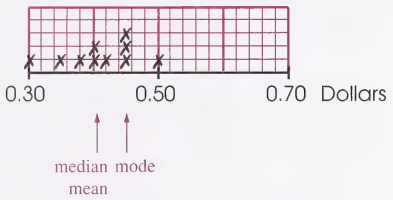
c. The mean price of bread in 1970 is \$0.41.

$$(0.30+0.35+0.38+2\times0.40+0.42+3\times0.45+0.50)\div10=0.41$$

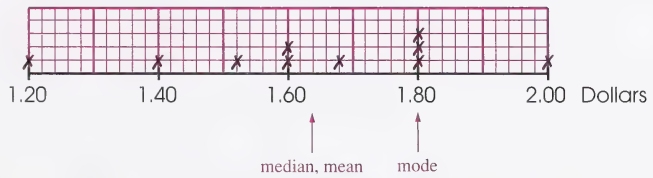
The mean price of bread in 1997 is \$1.64.

$$\frac{1.20+1.40+1.52+2\times1.60+1.68+3\times1.80+2.00}{10}=1.64$$

d. Following is the line plot for 1970.



e. Following is the line plot for 1997.



f. The data has been stretched. The mode, median, and mean in 1997 are 4 times the mode, median, and mean in 1970.

8. The set of data after the cut follows.

10 000	10 500	11 500	12 000	12 500
12 500	12 500	14 500	15 000	15 000

a. The mode before the cut is \$25 000.

The mode after the cut is \$12 500.

b. The mean salary before the cut is \$25 200.

$$(20\,200+21\,000+23\,000+24\,000+3\times25\,000+29\,000+2\times30\,000)\div10=25\,200$$

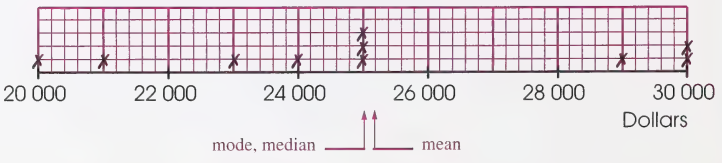
The mean salary after the cut is \$12 600.

$$(10\,000+10\,500+11\,500+12\,000+3\times12\,500+14\,500+2\times15\,000)\div10=12\,600$$

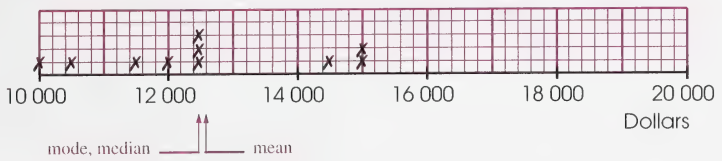
c. The median salary before the cut is \$25 000.

The median salary after the cut is \$12 500.

d. Following is a line plot of the salaries before the cut.



e. Following is a line plot of the salaries after the cut.



f. The data has been shrunk. The mode, median, and mean after the cut are one-half the mode, median, and mean before the cut.

9. Following is the set of data at the end of the year.

120 600 600 780 840

a. The mode value at the beginning of the year was \$500.

The mode value at the end of the year is \$600.

b. The median value at the beginning of the year is \$500.

The median value at the end of the year is \$600.

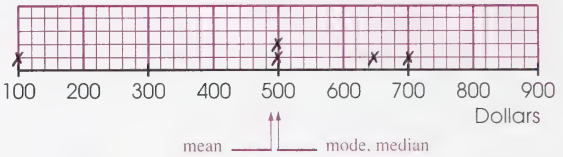
c. The mean value at the beginning of the year is \$490.

$$(100 + 2 \times 500 + 650 + 700) \div 5 = 490$$

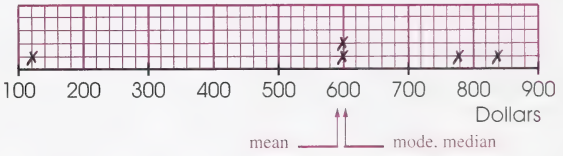
The mean value at the end of the year is \$588.

$$(120 + 2 \times 600 + 780 + 840) \div 5 = 588$$

d. Following is a line plot of the data at the beginning of the year.



e. Following is a line plot of the data at the end of the year.



f. The data has been stretched. The mode, median, and mean at the end of the year are 120% of the mode, median, and mean at the beginning of the year.

10. Following is the set of data during the sale.

30	32	38	38	40	40	40	44
52	52	52	52	52	56	60	

a. The mode price before the sale is \$78.
The mode price after the sale is \$52.

b. The median price before the sale is \$66.
The median price during the sale is \$44.

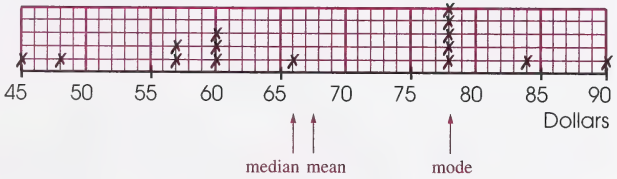
c. The mean price before the sale is \$67.80.

$$(45+48+2\times57+3\times60+66+5\times78+84+90)$$
$$\div 15 = 67.80$$

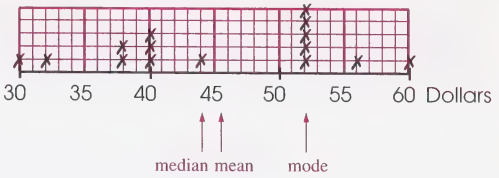
The mean price during the sale is \$45.20.

$$(30+32+2\times38+3\times40+44+5\times52+56+60)$$
$$\div 15 = 45.20$$

d. Following is a line plot of the data before the sale.



e. Following is a line plot of the data during the sale.



f. The data has been shrunk. The mean, median, and mode during the sale are two-thirds of the mean, median, and mode before the sale.

11. $26+2=28$

The new mean is 28 students per classroom.

$29+2=31$

The new mean is 31 students per classroom.

12. $2\times4.94=9.88$

The new mean is \$9.88/kg.

$2\times4.89=9.78$

The new median is \$9.78/kg.

13. $39+5=44$

The new mean is 44 years.

$18+5=23$

The new median is 23 years.

14. **a.** $(18\,000+20\,000+3\times22\,000+5\times24\,000+4\times25\,000$
$+28\,000+4\times30\,000+32\,000+34\,000+2\times35\,000$
$+36\,000+38\,000)\div25=27\,280$

The mean of the employees' salaries is \$27 280.

b. The median salary of the employees is \$25 000.

c. The median salary of the employees and owner is \$25 000.

d. The mode salary of the employees is \$24 000.

e. The mode salary of the employees and owner is \$24 000.

f. The mean is affected the most.

g. The statistic which best describes the average salary is the median or the mode.

15. a. The mode of Warren's weekly earnings before subtracting the start-up costs is \$300.

The mode of Warren's weekly earnings after subtracting the start-up costs is \$300.

b. The median weekly earnings before subtracting the start-up costs is \$290.

$$\frac{280+300}{2}=290$$

The median weekly earnings after subtracting the start-up costs is \$290.

c. $(150+175+220+280+3\times300+325)\div8=256.25$

The mean weekly earnings before subtracting the start-up costs is \$256.25.

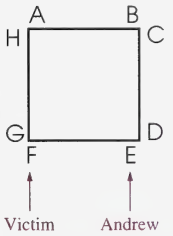
$$(-150+175+220+280+3\times300+325)\div8=218.75$$

d. The mean is affected the most.

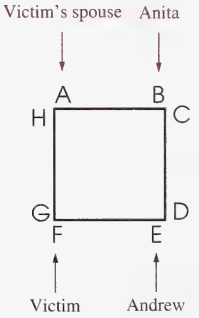
e. The median or the mode best represents Warren's average earnings.

16. Use logical reasoning and diagrams to solve the problem.

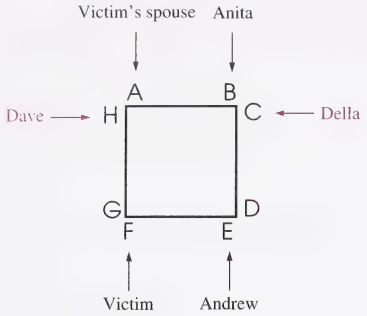
Step 1: The victim sat in chair F. The host sat on the victim's right. Therefore, Andrew sat in chair E.



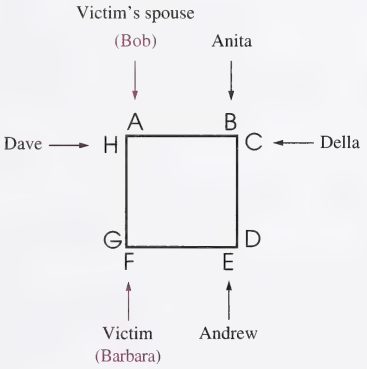
Step 2: Each man sat across from his wife. Therefore, the victim's spouse sat in chair A. Anita sat in chair B.



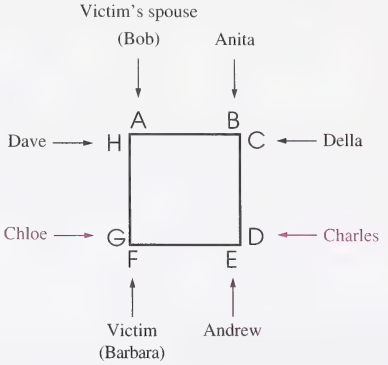
Step 3: Dave sat beside the victim's spouse. Therefore, Dave sat in chair H and his wife Della sat in chair C.



Step 4: Bob sat next to the hostess. Therefore, Bob sat in chair A and his wife Barbara sat in chair F.



Step 5: Each man sat between a man and a woman. Therefore, Charles sat in chair D and his wife Chloe sat in chair G.



Bob sat in chair A, Anita sat in chair B, Della sat in chair C, Charles sat in chair D, Andrew sat in chair E, Barbara sat in chair F, Chloe sat in chair G, and Dave sat in chair H.

Barbara was the victim.

Section 2: Activity 4

1. Let t represent the total number of haircuts.

$$\text{mean} = \frac{\text{total number of haircuts}}{\text{number of days}}$$

$$12 = \frac{t}{20}$$

$$20(12) = 20\left(\frac{t}{20}\right)$$

$$240 = t$$

The total number of haircuts that Frank did in July was 240.

2. Let t represent the total amount (in $) earned in tips.

$$\text{mean} = \frac{\text{total amount earned in tips}}{\text{number of tables served}}$$

$$2 = \frac{t}{110}$$

$$110(2) = 110\left(\frac{t}{110}\right)$$

$$220 = t$$

The total amount that the server earned in tips last week was $220.

3. Let t represent the total amount of wages (in $) per hour.

$$\text{mean} = \frac{\text{total amount of wages paid per hour}}{\text{number of employees}}$$

$$6 = \frac{t}{30}$$

$$30(6) = 30\left(\frac{t}{30}\right)$$

$$180 = t$$

The total amount of wages the owner pays per hour is $180.

4. **Step 1:** Calculate the total percentage earned on five tests.

Let t represent the total of the percentages earned on five tests.

$$\text{mean} = \frac{\text{total earned}}{\text{number of tests}}$$

$$78 = \frac{t}{5}$$

$$5(78) = 5\left(\frac{t}{5}\right)$$

$$390 = t$$

The total of the percentages that Maury got on five tests was 390%.

Step 2: Calculate the total of the percentages that Maury must get on six tests to raise his average to 80%.

Let t represent the total on six tests.

$$\text{mean} = \frac{\text{total earned}}{\text{number of tests}}$$

$$80 = \frac{t}{6}$$

$$6(80) = 6\left(\frac{t}{6}\right)$$

$$480 = t$$

To reach his target, the total of the percentages that Maury must get on six tests must be of at least 480%.

Step 3: Calculate the least percentage Maury must earn on the next test.

$$480 - 390 = 90$$

To reach his target, Maury must earn at least 90% on the next test.

5. a. Step 1: Calculate the total sales (in $) Janice will need to make in six days.

Let t represent the total.

$$\text{mean} = \frac{\text{total sales}}{\text{number of days}}$$

$$8500 = \frac{t}{6}$$

$$6(8500) = 6\left(\frac{t}{6}\right)$$

$$51\,000 = t$$

In the six days, Janice must make $51 000 in sales.

Step 2: Calculate how much Janice has already sold.

$$7530 + 8475 + 6550 + 7155 = 29\,710$$

In four days, Janice has sold $29 710.

Step 3: Calculate how much Janice must make in sales on Friday and Saturday.

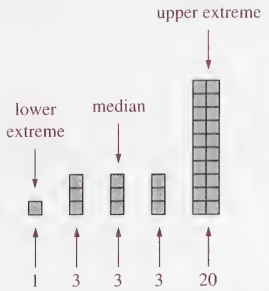
$$51\,000 - 29\,710 = 21\,290$$

To reach her target, Janice must make a minimum of $21 290 in sales on Friday and Saturday.

b. Answers will vary. Here is a possible answer.

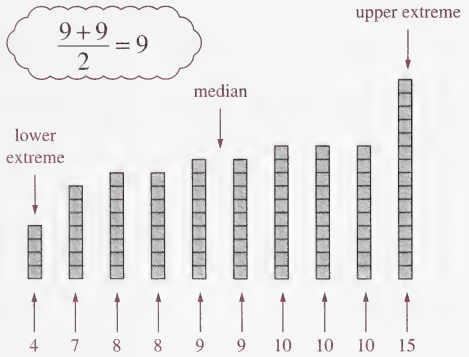
It may be unreasonable for Janice to sell almost as much in two days as she did in four days.

6. a. This is another solution to Example 2.



A set of data that has the given statistics is \$1, \$3, \$3, \$3, and \$20.

b. This is another solution for Example 3.



A set of data that has the given statistics is 4, 7, 8, 8, 9, 9, 10, 10, 10, and 15.

7. Step 1: Calculate the total number of goals scored.

Let t represent the total.

$$\text{mean} = \frac{\text{total}}{\text{number of games}}$$

$$3 = \frac{t}{6}$$

$$6(3) = 6\left(\frac{t}{6}\right)$$

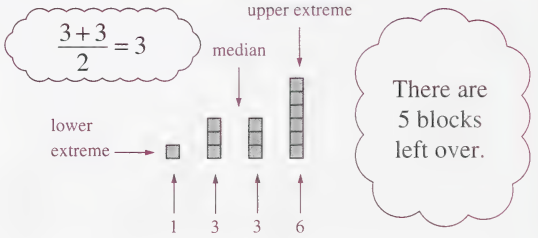
$$18 = t$$

The total number of goals scored is 18.

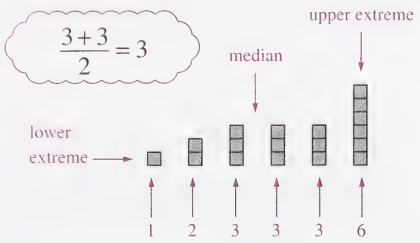
Step 2: Gather blocks to represent the total number of goals scored.

You will need 18 blocks.

Step 3: Use some of the 18 blocks to represent the median and the extremes of the 6 games.

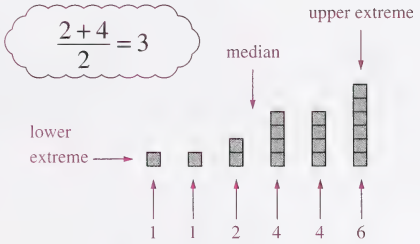


Step 4: Keeping in mind the median is 3 goals, use the remaining 5 blocks to reconstruct the number of goals scored per game in the other 2 games. Answers will vary. Here is one possible solution.



A set of data that has the given statistics is 1, 2, 3, 3, 3, and 6.

Note: This is another possible solution.



A set of data that has the given statistics is 1, 2, 4, 4, and 6.

8. Step 1: Calculate the total number of points.

Let t represent the total number of points.

$$\text{mean} = \frac{\text{total number of points}}{\text{number of students}}$$

$$5 = \frac{t}{13}$$

$$13(5) = 13\left(\frac{t}{13}\right)$$

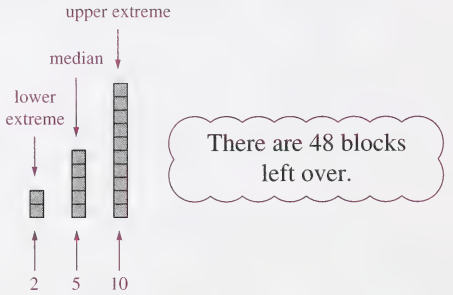
$$65 = t$$

The total number of points scored is 65.

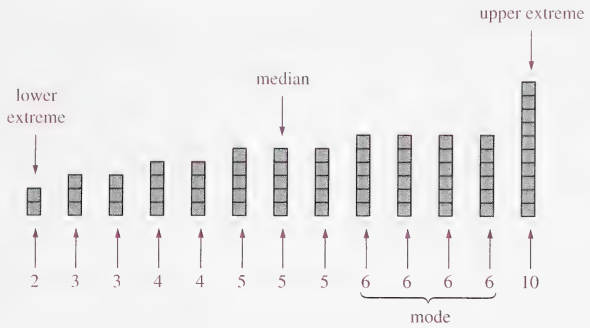
Step 2: Gather blocks to represent the total number of points scored by the 13 students.

You will need 65 blocks.

Step 3: Use some of the 65 blocks to represent the median and the extremes.

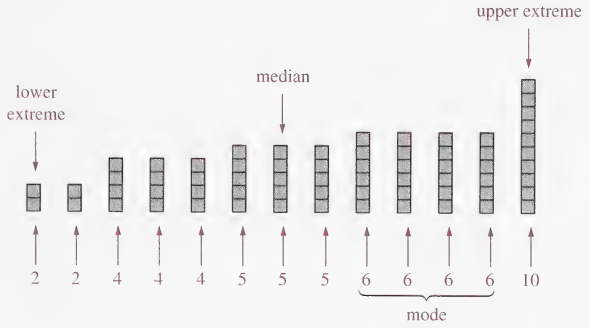


Step 4: Keeping in mind that the median is 5 and the mode is 6, use the remaining 48 blocks to reconstruct the scores of the other 10 students. Here is one solution.



A set of data that has the given statistics is 2, 3, 3, 4, 4, 5, 5, 5, 6, 6, 6, 6, and 10.

Note: This is another possible solution.



A set of data that has the given statistics is 2, 2, 4, 4, 4, 5, 5, 5, 6, 6, 6, 6, and 10.

9. Step 1: Calculate the total of the shoe sizes.

Let t represent the total of the shoe sizes.

$$\text{mean} = \frac{\text{total}}{\text{number of shoes}}$$

$$9 = \frac{t}{8}$$

$$8(9) = 8\left(\frac{t}{8}\right)$$

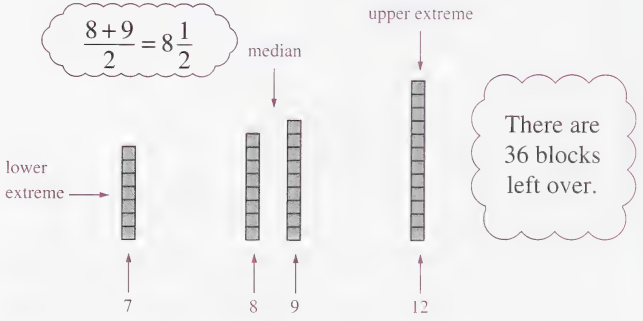
$$72 = t$$

The total of the shoe sizes is 72.

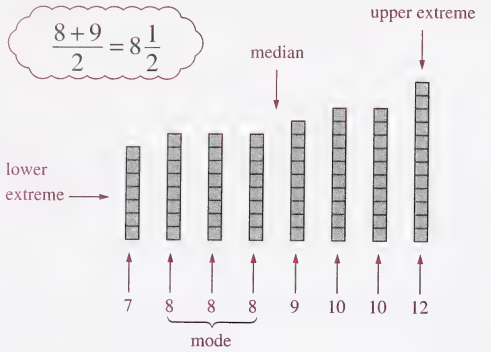
Step 2: Gather blocks to model the total of the shoe sizes of the 8 pairs of shoes.

You will need 72 blocks.

Step 3: Use some of the 72 blocks to model the median and the extremes.

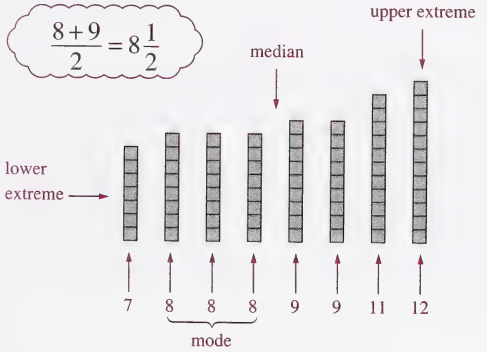


Step 4: Keeping in mind the mode is 8 and the median is $8\frac{1}{2}$, use the remaining 36 blocks to model the shoe sizes of the other 4 pairs of shoes. Here is one solution.



A set that has the given statistics is 7, 8, 8, 8, 9, 10, 10, and 12.

Note: This is another possible solution.



A set that has the given statistics is 7, 8, 8, 8, 9, 9, 11, and 12.

Now Try This

10. Use logical reasoning to solve this problem.

Step 1: Calculate the total price of the 5 items.

Let t represent the total.

$$\text{mean} = \frac{\text{total price}}{\text{number of items}}$$

$$4.40 = \frac{t}{5}$$

$$5(4.40) = 5\left(\frac{t}{5}\right)$$

$$22.00 = t$$

The total price of the 5 items is \$22.00.

Step 2: Calculate the total price of the 10 items.

Let t represent the total.

$$\text{mean} = \frac{\text{total price}}{\text{number of items}}$$

$$9.80 = \frac{t}{10}$$

$$10(9.80) = 10\left(\frac{t}{10}\right)$$

$$98.00 = t$$

The total price of the 10 items is \$98.00.

Step 3: Find the total price of the 15 items.

$$22.00 + 98.00 = 120.00$$

The total price of the 15 items is $120.00

Step 4: Find the mean price of the 15 items.

$$120.00 \div 15 = 8$$

The mean price is $8.

11. Use logical reasoning to solve this problem.

Step 1: Calculate the total.

Let t represent the total.

$$\text{mean} = \frac{\text{total}}{\text{number of tests}}$$

$$88 = \frac{t}{5}$$

$$5(88) = 5\left(\frac{t}{5}\right)$$

$$440 = t$$

The total of the five percentages is 440%.

Step 2: Calculate the highest total percentage Mark could have scored on 4 tests.

The highest total Mark could have scored on 4 tests was 400%.

$4 \times 100 = 400$

Step 3: Calculate the lowest percentage Mark could have scored on one test.

The lowest Mark could have scored on one test was 40%.

$440 - 400 = 40$

Section 2: Activity 5

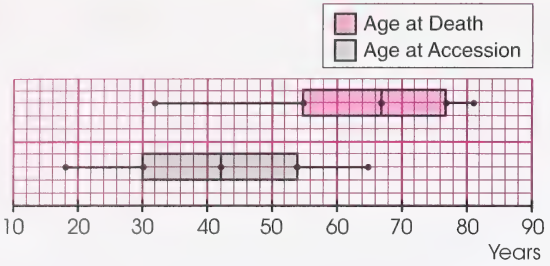
1. a. Each piece of data is shown in the double stem and leaf plot. You can see the clusters and gaps in each set of data.

b. The double box and whisker plot shows only certain statistics. It helps you visualize the extremes, quartiles, and medians of each set of data.

2. a.

Age at Accession		Age at Death
	0	
8	1	
7, 2	2	
9, 7, 0	3	2
5, 4, 2, 1	4	9
8, 4, 1	5	1, 5, 7
5, 0	6	7, 7, 7, 8
	7	0, 1, 7, 7
	8	1, 1

b.



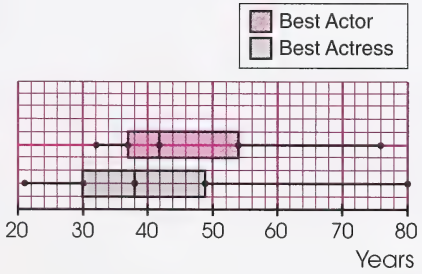
c. the age-at-death data **d.** the age-at-accession data

e. the age-at-death data

3. a.

Best Actress		Best Actor
9, 6, 1	2	
8, 6, 3, 3, 0	3	2, 3, 5, 7, 7, 8, 9
9, 6, 2, 1	4	2, 5
	5	1, 2, 4, 5
1	6	1
4	7	6
0	8	

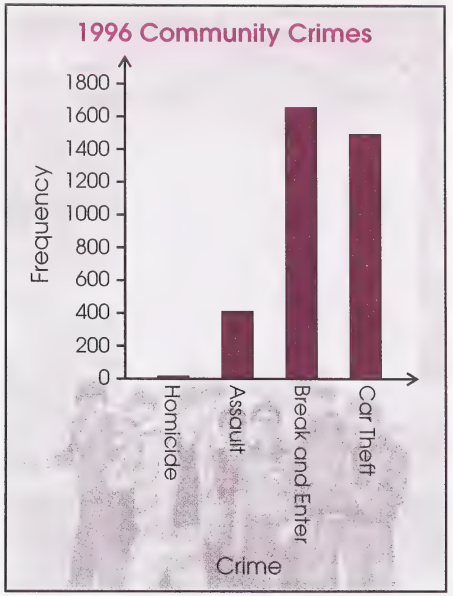
b.



c. the best-actress data **d.** the best-actress data

e. the best-actor data

4. a. Your graph should look similar to this.



b. **Step 1:** Use proportional reasoning to calculate how many degrees of the circle graph each crime should be.
Note: There are 3600 crimes. A circle has 360°.

Homicide

$$\frac{40}{3600} = \frac{4}{360}$$

Assault

$$\frac{390}{3600} = \frac{39}{360}$$

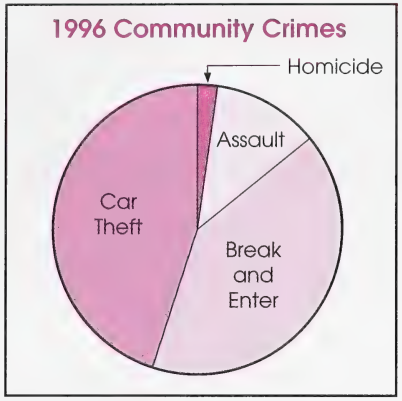
Break and Enter

$$\frac{1670}{3600} = \frac{167}{360}$$

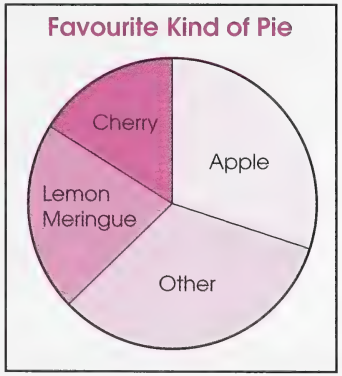
Car Theft

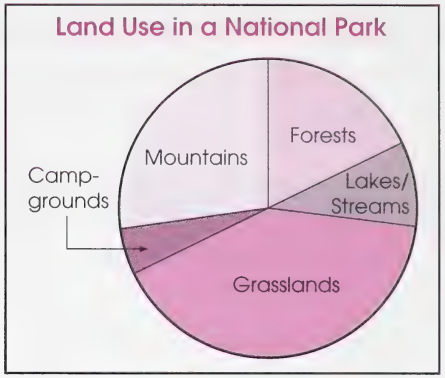
$$\frac{1500}{3600} = \frac{150}{360}$$

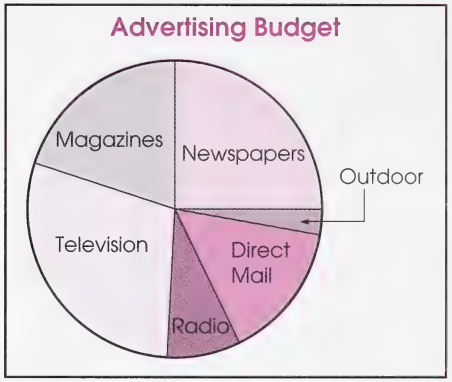
Step 2: Draw a circle graph with a compass and straightedge. Label the graph. Your graph should look similar to this.



5. Your graphs should look similar to the following.





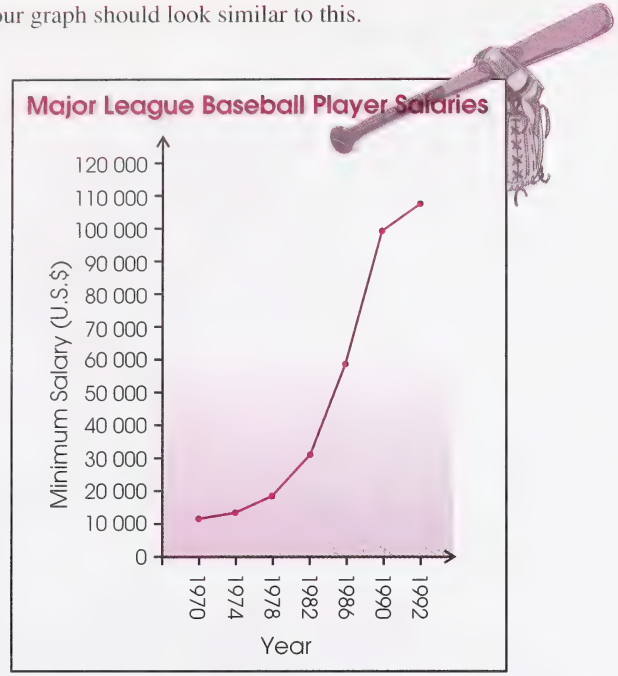


What do you call a person who buys and sells bugs?

AN ANT TICK DEALER

Note: There are no letters for 101°, 62°, and 16°.

6. Your graph should look similar to this.



7. **a.** Sarah entered the following headings and data on a new spreadsheet file using *ClarisWorks*™.

	A	B
1	Alberta Revenue 1992/93	
2	Taxation	4695000000
3	Gov't of Canada	2384000000
4	Non-Renewable Resource Revenue	2183000000
5	Investment Income	1621000000
6	Fees, Permits, and Licences	1027000000
7	Other	1314000000

b. To make the circle graph, Sarah highlighted cells A2 to B7, selected "Options" on the menu bar and clicked on "Make Chart." In the Chart Options box, Sarah selected "Pie" (another name for a circle graph), selected "Shadow," and clicked on "OK."

To put the title on the graph, Sarah selected "Options" on the menu bar and dragged down to "Modify Chart." In the Chart Options box, Sarah selected "Labels," entered the title, selected "Shadow" for the title and legend, and clicked on "OK."

To put labels on the sectors of the circle graph, Sarah selected "Options" on the menu bar and dragged down to "Modify Chart." In the Chart Options box, Sarah selected "Series." Then she selected "Label data" as "% in slice" and clicked on "OK."

To highlight one sector, Sarah clicked on the box beside the appropriate sector (non-renewable resource revenue) in the legend of the graph. Then she selected "Options" in the menu bar and dragged down to "Modify Chart." In the Chart Option box, Sarah selected "Explode slice" and clicked on "OK."

Finally Sarah saved the file and printed the graph.

8. **a.** Michael entered the following headings and data on a new spreadsheet file using *ClarisWorks*™.

	A	B	C	D	E	F
1	Population	'51	'61	'71	'81	'91
2	East	1618126	1897425	2057264	2234035	2322081
3	Central	8653223	11495303	13730871	15063512	16980848
4	West/North	3738080	4845519	5780185	7045640	7990930

b. To make the bar graph, Michael highlighted cells A1 to F4, selected "Options" on the menu bar, and dragged down to "Make Chart." In the Chart Options box, Michael selected "Bar" and clicked on "OK."

Michael adjusted the size of the bar graph and the scale on the *y*-axis by clicking on the handles of the graph box and stretching the box.

To label the *x*-axis and remove the grid lines, Michael selected "Options" on the menu bar and dragged down to "Modify Chart." In the Chart Options box, Michael selected "Axis." Then Michael selected "*x*-axis," entered the axis label, deselected grid lines, chose "none" for the tick marks, and clicked on "OK."

To label the *y*-axis and remove the grid on the *y*-axis, Michael selected "Options" on the menu bar and dragged down to "Modify Chart." In the Chart Options box, Michael selected "Axis." Then Michael selected "*y*-axis," entered the axis label, deselected "Grid lines," and clicked on "OK."

To add the title and bar labels, Michael selected "Options" on the menu bar, and dragged down to "Modify Chart." In the Chart Options box, Michael selected "General." Then Michael selected "series in rows" and "use numbers as labels in first row." He then pressed "Return" or "OK."

Finally Michael saved the file and printed the graph.

c. To make the broken-line graph, Michael opened the file he created for the bar graph, highlighted cells A1 to F4, selected "Options" on the menu bar, and clicked on "Make Chart." In the Chart Options box Michael selected "Line" and clicked on "OK."

Michael adjusted the size of the broken-line graph and the scale on the *y*-axis by clicking on the handles of the graph box and stretching the box.

To label the *x*-axis and *y*-axis, remove the grid lines, remove ticks on the *x*-axis, add the title, and label the years, Michael followed the same steps as he did for the bar graph. (See the answer to question 16.b.). To change the symbols on the lines, Michael selected "Options" on the menu bar, and dragged down to "Modify Chart." In the Chart Options box, Michael selected "Series." Then Michael chose the filled dot as the symbol and pressed "Return" or "OK."

Finally Michael saved the file and printed the graph.

9. a. Rosemary entered the following headings and data on a new spreadsheet file using *ClarisWorks*™.

	A	B	C
1	Vehicle Production	Passenger	Commercial
2	United States	6613970	12262737
3	Japan	7801317	2752802
4			

b. To make the graph, Rosemary highlighted cells A1 to C3, selected "Options" in the menu bar, and clicked on "Make Chart." In the Chart Options box, Rosemary selected "Pie" and clicked on "OK."

To label the graph, Rosemary selected "Options" in the menu bar and clicked on "Modify Chart." In the Chart Options box, she selected "Labels." She also selected "horizontal" for the legend. Then she clicked on "OK."

To label the sectors on the circle graph, Rosemary selected "Options" on the menu bar and clicked on "Modify Chart." In the Chart Options box, Rosemary selected "Series." Then she selected "Label data" as "% in slice" and clicked on "OK."

To have the graph show vehicle production by country, Rosemary selected "Options" on the menu bar, and dragged down to "Modify Chart." In the Chart Options box, Rosemary selected "General." Then Rosemary chose "Columns" under "Series in" and pressed "Return" or "OK."

Rosemary adjusted the size of the graph by clicking on the handles of the graph box and stretching the box.

Finally Rosemary saved the file and printed the graph.

Now Try This

10. Use patterns to help you solve the problem.

Step 1: Find the time indicated on the watch at noon on June 6.
Note: The watch loses 8 min in 24 h.

Date	Correct Time	Time Indicated on Watch	
June 1	12:00	12:00	
June 2	12:00	11:52	−8 min
June 3	12:00	11:44	−8 min
June 4	12:00	11:36	−8 min
June 5	12:00	11:28	−8 min
June 6	12:00	11:20	−8 min

At noon on June 6, the time indicated on the watch will be 11:20.

Step 2: Use proportional reasoning to find the time lost in 6 h.

$$\frac{8}{24} \overset{\div 4}{=} \frac{2}{6}$$

If the watch loses 8 min in 24 h, it will lose 2 min in 6 h.

Step 3: Find the time indicated on the watch at 18:00 on June 6.

Date	Correct Time	Time Indicated on Watch
June 6	12:00	11:20
June 6	18:00	17:18

+6 h; −2 min

At 18:00 on June 6, the time indicated on the watch will be 17:18.

Section 2: Follow-up Activities

Extra Help

1. **Step 1:** Calculate the total number of baskets that Randy scored in 6 games.

   Let t represent the total number of baskets.

   $$\text{mean} = \frac{\text{total baskets}}{\text{number of games}}$$

   $$5 = \frac{t}{6}$$

   $$6(5) = 6\left(\frac{t}{6}\right)$$

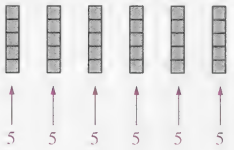
   $$30 = t$$

   The total number of baskets that Randy scored is 30.

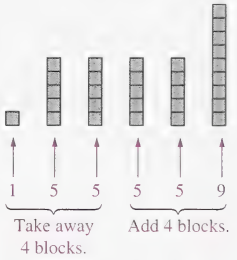
Step 2: Gather blocks to represent the total number of baskets.

You will need 30 blocks.

Step 3: The mean is 5. Therefore, arrange the 30 blocks in 6 stacks of 5.

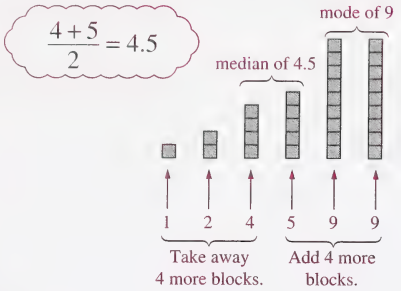


Step 4: The least number of baskets that Randy scored is 1 and the most is 9. Therefore, make the adjustments keeping in mind that balance must be maintained.



Step 5: The mode is 9 and the median is 4.5. Therefore, make the adjustments keeping in mind that balance must be maintained.

Here is the solution.



A set of data that has the given statistics is 1, 2, 4, 5, 9, and 9.

2. **Step 1:** Calculate the total number of vibrations per second by the 10 waterfalls.

Let t represent the total number of vibrations per second.

$$\text{mean} = \frac{\text{total vibrations per second}}{\text{number of waterfalls}}$$

$$16 = \frac{t}{10}$$

$$10(16) = 10\left(\frac{t}{10}\right)$$

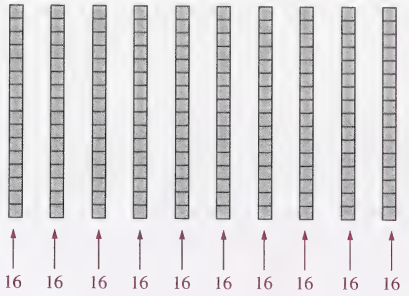
$$160 = t$$

The total number of vibrations per second is 160.

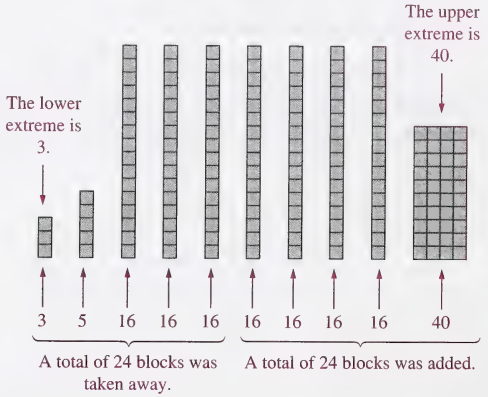
Step 2: Gather blocks to represent the total number of vibrations per second.

You will need 160 blocks.

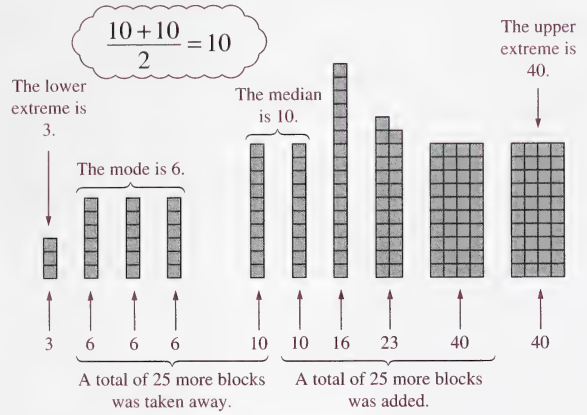
Step 3: The mean is 16. Therefore, arrange the 160 blocks in 10 stacks of 16.



Step 4: The least number of vibrations per second is 3; the greatest number is 40. Therefore, make the adjustments keeping in mind that the balance must be maintained.


The lower extreme is 3.
The upper extreme is 40.
3 5 16 16 16 16 16 16 16 40
A total of 24 blocks was taken away.
A total of 24 blocks was added.

Step 5: The mode is 6. The median is 10. Therefore, make the adjustments keeping in mind that the balance must be maintained.


$\frac{10+10}{2}=10$
The lower extreme is 3.
The mode is 6.
The median is 10.
The upper extreme is 40.
3 6 6 6 10 10 16 23 40 40
A total of 25 more blocks was taken away.
A total of 25 more blocks was added.

A set of data that has the given statistics is 3, 6, 6, 6, 10, 10, 16, 23, 40, and 40.

Enrichment

1. a. About one-half of the scores are between 6 and 8.

b. About one-quarter of the scores are 8 or above.

c. About one-quarter of the scores are 6 or below.

d. You can see that the majority of the scores are between 5 and 9.

2. a.

Stem	Leaf
1	5, 6, 8
2	0, 1, 1, 2, 4, 5, 5, | 6, 8
3	2, 4, 4, 4
4	1, 2, 5
5	9

$\frac{25+26}{2}=25.5$

The median is \$25.50.

b.

Stem	Leaf
1	5, 6, 8
2	0, 1, | 1, 2, 4, 5, 5, | 6, 8
3	2, 4, 4, | 4
4	1, 2, 5
5	9

The lower quartile is \$21. The upper quartile is \$34.

c. The lower extreme is \$15. The upper extreme is \$59.

d. $34-21=13$

The interquartile range is \$13.

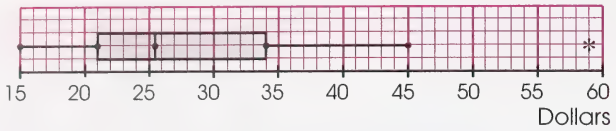
$1.5\times13=19.50$

Any outliers will be more than \$19.50 below the lower quartile or more than \$19.50 above the upper quartile.

$21.50-19.50=2;\ 34.00+19.50=53.50$

There is one outlier—a cost of \$59.

e.



3. a. The lower extreme is 2%. The upper extreme is 65%.

b. $65 - 2 = 63$

The range is 63%.

c. The median is 40%.

d. The lower quartile is 35%. The upper quartile is 45%.

e. $45 - 35 = 10$

The interquartile range is 10%.

f. There are four outliers—a cereal that is 2% sugar, one that is 8% sugar, one that is 15% sugar, and one that is 65% sugar.

g. $1.5 \times 10 = 15\%$

The outliers indicate cereals that are more than 15% above the upper quartile or more than 15% below the lower quartile.

Most of the 30 kinds of cereals are between 28% sugar and 58% sugar.

4. a.

−30	−25	−9	−5	−4	−4
−2	0	3	5	15	22

The median is -3°C.

b.

−30	−25	−9	−5	−4	−4
−2	0	3	5	15	20

The lower quartile is -7°C. The upper quartile is 4°C.

c. The lower extreme is -30°C. The upper extreme is 22°C.

d. $4 - (-7) = 11$

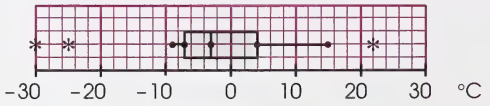
The interquartile range is 11°C.

$1.5 \times 11 = 16.5$

Any outliers will be more than 16.5°C below the lower quartile or more than 16.5°C above the upper quartile.

$-7 - 16.5 = -23.5;\ 4 + 16.5 = 20.5$

There are three outliers—temperatures of -25°C, -30°C, and 22°C.



The Web Barrier

Invention: The "web barrier," a starting gate used in thoroughbred racing.

Date: Ca. 1890

Inventor: Philip McGinnis was born at Athelstan, Quebec, south of Montreal. He started out as a newspaper reporter before turning to invention. McGinnis died Nov. 25, 1937.

History: For centuries, the horsey set had sought an efficient way to start races. In 18th-century England, a drum was used to signal starts. The drummer began with a long roll, and, as he ended with a loud beat, the horses set off. The method was transplanted to Canada and the United States, where it was replaced by the starter's pistol. That method proved hard on the nerves of horses and spectators alike, and was quickly abandoned. A starting flag provided a quieter alternative, but resulted in many delays. In one instance, at the 1898 Futurity, the race was delayed 90 minutes because of false starts.

By the end of the 19th century, the search was on for a modern, reliable method of beginning horse races. Records from the period are confusing, and apparently several individuals were experimenting with various types of gates around this time. Among them was Philip McGinnis, who covered the racing beat for the New York *Sun*. According to one source, August Belmont, the head of the New York Jockey Club, encouraged McGinnis to construct a reliable starting gate. Although he had no formal engineering training, McGinnis was a creative and ambitious young man. He designed a prototype which was built at a Brooklyn machine shop and quickly became a success.

The gate was basically a web made of strands of wire, lowered across the race track in a spring-levered device. With the horses lined up behind it, all the starter needed to do was touch the lever. The barrier flew up and the horses were off. McGinnis's web barrier was apparently a vast improvement over the "Australian barrier," an apparatus of heavy ropes, strung horizontally and prevented from tangling by metal spreaders. Although it operated in basically the same way as McGinnis's gate, the Australian barrier was heartily disliked by both starters and jockeys, especially after one impatient rider was caught in the ropes and seriously injured.

Buoyed by the success of his invention, McGinnis abandoned his newspaper career and moved to London, Ontario, where he set up a business to manufacture his gate. Despite his lack of training, he produced his own drawings, designs and patterns, which were cast in Owen Sound. The McGinnis children, four sons and two daughters, often helped out in the business during peak periods. McGinnis, who listed his occupation as "inventor" in the London city directory, spent the remainder of his life promoting his invention. The web barrier was patented in Canada, the U.S. and abroad, and made its inventor wealthy. When he was not actually producing gates, McGinnis travelled to race tracks in Canada and the States to publicize its merits and officiate at various racing events.

The arrival of the 20th century made it inevitable that the web barrier would give way to an improved, electrical version. Late in life, Philip McGinnis was approached by investors to design a stall-type gate. He refused, feeling it unfair to confine high-spirited thoroughbreds in such a device. So it was left to other inventors, including some Canadians, to develop the stall-type starting gate in use today.[1]

[1] Cheryl MacDonald, "Discoveries and Inventions," *Horizon Canada*, vol. 7, no. 75 (1985): third cover.

Plastic Garbage Bags

Date: 1960s

Inventor: Frank Plomp, Toronto entrepreneur, born in 1921.

Significance: Plastic garbage bags have replaced traditional galvanized iron garbage cans because they offer a cleaner and more efficient way to collect municipal waste.

Profile: Entrepreneurs like Frank Plomp do not invent new technologies but rather they exploit potential applications of such technologies. In Plomp's case, the technology involved plastic fabrication—in particular, the technique of welding thin polyethylene film to form a tough bag with strong seams. And the potential application was bagging garbage.

Plomp, who was president of a Toronto envelope company, formed his own company to develop and market what he called the Garbag. In 1959, the F.G. Plomp Company began selling bags made of clear, polyethylene film to hospitals and offices, as liners for wastebaskets and garbage cans. They were adopted because they saved money. At IBM headquarters in Don Mills, for instance, the use of Plomp's bags meant garbage cans no longer had to be steam-cleaned every three months.

At the beginning of 1962, Plomp decided to expand from the industrial into the residential market. His new bags were made on a specially-designed welding machine—it could switch from welding the ends of bags to the sides in only four minutes—by W. Ralston and Company, a plastic-extruding firm in Toronto. The bags were green in colour, so that they would blend discreetly with suburban lawns, and opaque, to hide their contents.

The battle for Canada's garbage had begun.

Bags or Cans? Before the 1960s, Canadians put their garbage out in trash cans. On garbage day, the collectors would carry the full cans to a truck, and shake and thump them to empty them. If the wind was blowing, scraps, dust and ashes were scattered around. The garbage collectors then returned the empty cans, usually making a good deal of noise.

Slinging garbage packed in garbage bags, collectors can do the same amount of work in much less time, with less mess and less noise. Since the bags are tied, odours are sealed in, greatly reducing the numbers of insects, dogs and cats attracted by the garbage. Such were the advantages claimed by Plomp, and demonstrated by trials of garbage bags in the streets and dumps of Waterloo, Ont., in 1962.

Such trials were convincing. Garbage bags gradually replaced metal cans during the 1960s, first in small municipalities in Ontario, then, once by-laws had been amended to allow garbage to be bagged, in large cities such as Toronto and Montreal (which adopted them in time for Expo '67).[1]

[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 3, no. 31 (1985): third cover.

When the Boy Tire Maker Married the Girl Tire Maker, What Did Everyone Say?

Do each exercise and find your answer at the bottom of the page. Write the letter of the exercise in the box above the answer.

1. Suppose you roll a regular 6-faced die.

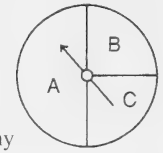
(A) How many equally likely outcomes are there?

(E) If you roll the die once, what is the probability of rolling a 3?

(H) If you roll the die 60 times, about how many times would you expect to get a 1?

(I) If you roll the die 300 times, about how many times would you expect to get a 5?

2. Suppose you have the following spinner for which each outcome is **not** equally likely.



(E) If you twirl the spinner once, what is the probability that it will stop on A?

(A) If you twirl the spinner once, what is the probability that it will stop on B?

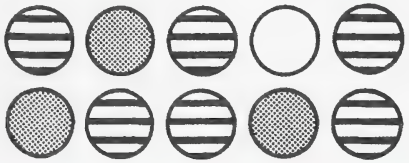
(T) If you twirl the spinner 50 times, about how many times would you expect it to stop on A?

(Y) If you twirl the spinner 80 times, about how many times would you expect it to stop on C?

3. Find each probability if you choose one marble at random.

(E) *P (black)* (S) *P (striped)* (A) *P (not black)*

(E) *P (not white)* (R) *P (black or white)* (M) *P (yellow)*



4. Solve the following problems.

(N) If you flip a coin 150 times, about how many times would you expect to get "Heads"?

(C) If one letter of the alphabet is chosen at random, what is the probability it is a vowel? **Note:** *y* may also be a vowel.

(K) If you randomly pick a date in April, how many equally likely outcomes are there?

(P) A friend asks you to pick a card, any card, from a standard deck of 52 cards. What is the probability of picking an ace?

25	10	$\frac{9}{10}$	20	$\frac{1}{8}$	0	6	30	$\frac{1}{2}$	$\frac{2}{13}$	$\frac{7}{10}$	$\frac{1}{5}$	75	50	$\frac{3}{13}$	$\frac{1}{6}$	32	$\frac{3}{5}$	$\frac{1}{13}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{3}{10}$

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What Do You Call a Person Who Buys and Sells Bugs?

Find the measure of each central angle (rounded to the nearest degree). Write the letter for each answer in the box containing the answer at the bottom of the page. Construct a circle graph for each set of data.

Favourite Kind of Pie (results of a survey)		
Kind	% of Total	Central Angle
Apple	30%	(C)
Cherry	16%	(A)
Lemon meringue	21%	(E)
Other	33%	(R)

Land Use in a National Park		
Land Use	% of Total	Central Angle
Forests	18%	(I)
Mountains	27%	(N)
Campgrounds	5%	(A)
Grasslands	41%	(T)
Lakes/Streams	9%	(E)

Advertising Budget		
Medium	% of Total	Central Angle
Newspapers	25%	(K)
Magazines	20%	(A)
Television	29%	(D)
Radio	8%	(N)
Direct Mail	15%	(L)
Outdoor	3%	(T)

72°	97°	101°	58°	29°	148°	62°	11°	65°	108°	90°	16°	104°	76°	18°	54°	32°	119°

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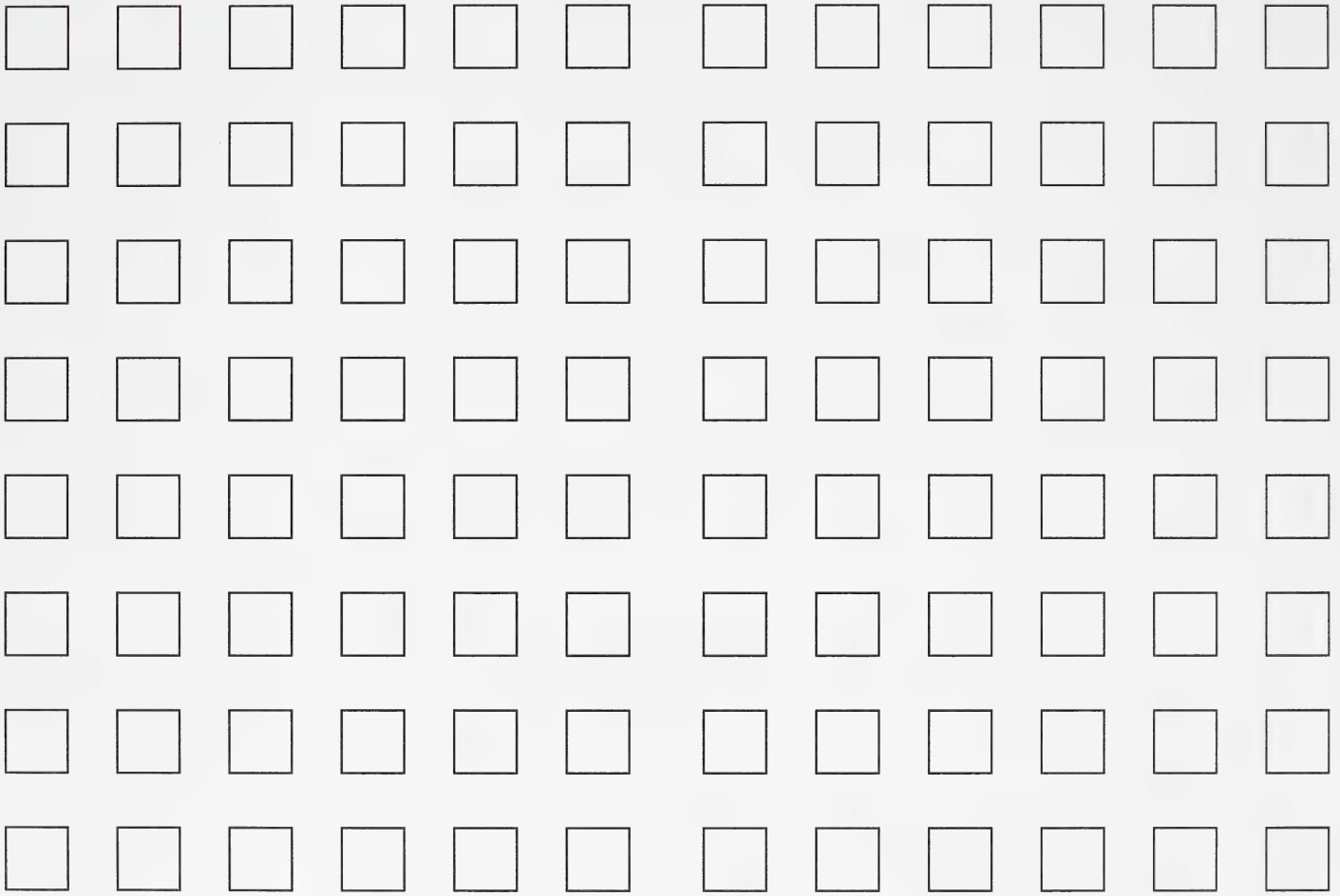
Base Ten Blocks



Student and teacher: Use this cover sheet for mailing or faxing.

ASSIGNMENT BOOKLET

8110 Mathematics 8
Module 5

FOR STUDENT USE ONLY

Date Module Submitted:

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File Number:

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Student's Questions and Comments

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Please verify that preprinted label is for correct course and module.

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Teacher

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- Are all the assignments completed? If not, explain why.
- Has your work been reread to ensure accuracy in spelling and details?
- Is the booklet cover filled out and the correct module label attached?

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Send all letters in a separate envelope.

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MATHEMATICS 8
MODULE 5

Statistics and Probability

ASSIGNMENT BOOKLET

Distance Learning

Alberta EDUCATION

FOR TEACHER'S USE ONLY

Summary

	Total Possible Marks	Your Mark
Section 1 Assignment	35	
Section 2 Assignment	40	
Final Module Assignment	25	
	100	

Teacher's Comments

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 8
Assignment Booklet
Module 5
Statistics and Probability
Learning Technologies Branch
ISBN 0-7741-1379-0

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ASSIGNMENT BOOKLET
MATHEMATICS 8 – MODULE 5: STATISTICS AND PROBABILITY

Your mark on this module will be determined by how well you do your assignments in this booklet.

Work slowly and carefully. If you are having difficulties, go back and review the appropriate section.

There are two section assignments and one final module assignment in this Assignment Booklet. The total value of these assignments is 100 marks. The value of each assignment is stated in the left margin.

Be sure to proofread each assignment carefully.

35

Section 1 Assignment: Chance and Uncertainty

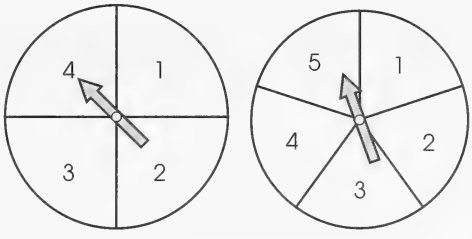
Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

1. Following are the rules for a game.

You will need the following:

- paper and pencil to keep score
- two dials with spinners

Note: One dial has 4 equal sectors and the other dial has 5 equal sectors. You can make these dials yourself using a compass and protractor. You can use a pencil and paper clip for each spinner.



Game Rules

- Choose one person to be "odd" and the other person to be "even." **Hint:** You may flip a coin to determine who will be the "odd" person.
- Take turns twirling each spinner and finding the product of the two numbers.
- If the product is an odd number, the "odd" person scores a point. If the product is an even number, the "even" person scores a point.
- The winner is the person with more points after 20 rounds.

(2) **a.** Play the game with a friend. Based on the results of the game, does the game appear to be fair? Explain.

(5) **b.** List all the possible products in the game.

(3) **c.** Calculate the theoretical probability of the product being an odd number.

(3) **d.** Calculate the theoretical probability of the product being an even number.

(2) **e.** Based on the theoretical probability, which person has the greater advantage—the person choosing "odd" or the person choosing "even"? Did your experimental results (see question 1.a.) support this conclusion?

_____

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2. Sandra shuffles a deck of 52 playing cards. Ruth then randomly picks a card from the deck. If aces are high, give the probability of each of the following outcomes. (**Hint:** You may use a standard deck of cards to help you identify the favourable outcomes, or you may use the pictures of the cards in the Student Module Booklet.)

(3) **a.** a card higher than a jack

(3) **b.** a black card with an even number (2, 4, 6, 8, or 10)

(2) **3.** A carpet-cleaning business telephones residences in a rural town in order to set up appointments. Records show that as the result of 200 telephone calls made, 36 appointments for carpet cleaning were arranged.

Predict the number of appointments that will be made from 500 calls.

(2) **4.** A manufacturer produced 50 000 hockey sticks in a month. The manufacturer tests 3000 of the hockey sticks and finds 6 defective sticks. How many of the 50 000 hockey sticks produced that month are likely to have been defective?

Use a computer to solve question 5. If you do not have access to a computer, use an appropriate probability tool.

(10)

5. A baseball player has a batting average of 0.250. This means that, on average, the player has been getting 1 hit out of every 4 trips to the plate. What is the probability that the player will get 2 or more hits in the next 5 times the player is up to bat?

Use the Monte-Carlo method to solve this problem. Explain clearly how you arrived at your answer and include all your data.

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Section 2 Assignment: Data Management

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

1. The following table lists six electrical devices and the average yearly expense (in dollars) for an Albertan family to operate each device. (These electrical devices use the majority of the electrical energy consumed by a household.)

Electrical Device	Yearly Expense ($)
Dryer	86
Furnace	393
Hot Water Heater	232
Interior Lights	70
Refrigerator	142
Stove	67

(3) **a.** Explain how these statistics may have been estimated.

(2) **b.** On average, which electrical device consumes the least electricity in a year and is therefore least expensive to operate?

(2) **c.** On average, which electrical device consumes the most electricity in the year and is therefore the most expensive to operate?

_____

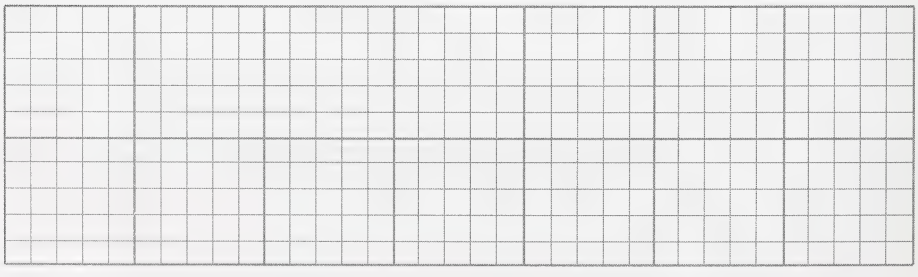
_____

(2) **d.** What is the range of the operating expenses?

(2) **e.** What is the mean cost per family to operate these electrical devices for a year?

(2) **f.** What is the median cost per family to operate these electrical devices for a year?

(3) **2.** Make a box and whisker plot of the data in question 1.



Use a computer and spreadsheet program to answer question 3. If you do not have access to a computer, you may plot the graphs by hand.

(5) **3. a.** Make a circle graph of the data in question 1.

(3) **b.** Make a bar graph of the data in question 1.

4. A city traffic department collected the following data on the number of vehicles that passed through a major intersection during different time periods between 06:00 and 21:00 on a particular day. **Note:** The numbers have been rounded to the nearest hundred.

Time	Number of Cars
06:00–07:00	500
07:00–08:00	700
08:00–09:00	200
09:00–10:00	100
10:00–11:00	100
11:00–12:00	100
12:00–13:00	100
13:00–14:00	100
14:00–15:00	100
15:00–16:00	100
16:00–17:00	700
17:00–18:00	600
18:00–19:00	100
19:00–20:00	100
20:00–21:00	50

(2) **a.** What is the mean number of cars per hour?

(2) **b.** What is the median number of cars per hour?

(2) **c.** What is the mode of the number of cars per hour?

(2) **d.** Which measure(s) of central tendency best describes the number of cars passing through the intersection? Explain.

(4)

5. A shoe salesperson sells 9 pairs of shoes one morning. The mean size of the shoes is 7. The mode of the sizes is 7. The median size is 7. The smallest shoe size is 5. The largest shoe size is 9. None of the shoes is a half size.

Reconstruct a set of data that has these statistics. **Hint:** You may use blocks to help you reconstruct the sizes. Explain how you arrived at your answer.

6. The number of passengers in different buses was recorded. The mean number of passengers on the buses is 46 and the median is 47.

(2) **a.** Each passenger paid $1.25 to ride the bus. What would be the mean amount of money collected? What would be the median amount of money collected?

(2) **b.** If 20 extra passengers rode on each bus, what is the new mean number of passengers on the buses? What is the new median?

(25)

Final Module Assignment

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

1. Sarah wants to know how much garbage is produced in the average home in her community.

(5) **a.** Design a simple questionnaire that Sarah could use to investigate this question.

(3) **b.** Explain how Sarah could carry out this survey.

(1) **c.** What statistics could Sarah use to describe the distribution of the data she collects?

(1) **d.** What statistics could Sarah use to describe the central tendency of the data she collects?

(1) **e.** What could Sarah do to help her visualize any clusters or gaps in the data?

2. State which measure of central tendency each of the following people would find most useful for the given purpose. Explain why this measure is most appropriate.

(1) **a.** a store owner deciding what sizes of shoes to order

(1) **b.** bowlers comparing the average scores of team members

(1) **c.** a teacher reporting the average score on a test

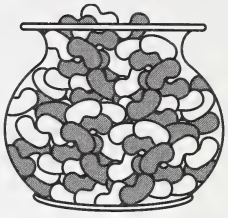
3. In a local or regional newspaper, find an article with data that is related to a current civic or regional issue (for example, elections, unemployment, housing, environmental issues). Include the newspaper clipping and answer the following questions.

(3) **a.** Are the data presented in a fair, clear, and appropriate manner? Explain.

(3) **b.** What questions about the data are addressed and not addressed?

(5)

4. A jar contains red and white jellybeans. There are 1000 jellybeans in all. Explain how you can draw samples from the jar and use the samples to predict the number of red jellybeans in the jar.



ASSIGNMENT BOOKLET DECLARATIONS

The Student's Declaration is to be filled in by a student registered at the Alberta Distance Learning Centre. If the student is under 16, the Learning Facilitator's Declaration is to be filled in by the learning facilitator. Failure to complete this page may invalidate the assignment results.

STUDENT'S DECLARATION

- I have followed the instructions outlined in the Student Module Booklet.
- I have completed the activities to prepare myself for the assignments in this Assignment Booklet.
- I completed the assignments in this Assignment Booklet by myself.

_____

Student's Signature

LEARNING FACILITATOR'S DECLARATION

I hereby certify that I have supervised the learning activities completed by _____.

Student's Name

I also certify that to the best of my knowledge the assignments in this Assignment Booklet were completed independently by this student.

_____

Supervisor's Signature

If you, the student or learning facilitator, have any comments or observations regarding this module, write them in the following space.

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